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Research Article

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Particle Swarm Optimization Based LQR Control of an Inverted Pendulum

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ARTICLE INFO	ABSTRACT
Corresponding Author: **Ümit Önen** Necmettin Erbakan University, Department of Mechatronics Engineering, Konya, Turkey	Development of new control methods and the improvement of existing control techniques have been interest of researchers for many years. Inverted pendulum systems have been used to test the performance of various control methods in many studies due to their unstable and nonlinear structures. In this work, the use of Particle Swarm Optimization algorithm is presented for the parameter optimization of a Linear Quadratic Regulator controller designed to stabilization and position control of an inverted pendulum. Equations of motion of the cart pendulum derived by Lagrange formulation have been linearized and presented as state-space model. A Linear Quadratic Regulator controller has been designed for position control and stabilization of pendulum system. Parameters of the controller have been optimized by Particle Swarm Optimization algorithm to obtain best control results. Simulation studies were carried out in the MATLAB/Simulink environment and performance of the designed controller has been evaluated through simulation results.

KEYWORDS:*Inverted pendulum, Position control, LQR control, PSO algorithm, Optimization.*

I. Introduction

For many years, researchers have been studying on the development of new control methods. One of the commonly used systems for examining the performance of control methods is the inverted pendulum. Inverted pendulum systems where the number of controlled variables is greater than the control input is ideal for testing the performance of control methods due to unstable and nonlinear structure. For this reason, inverse pendulums have been used in literature for many studies on modelling and control. Inverted pendulum was used as a simplified model of humanoid robot [1], hexapod robot [2] human body kinematics [3,4], vehicle dynamics [5].

When examined from the control point of view, there are many studies in the literature for controlling the inverted pendulum systems. PID, LQR and sliding mode controllers were successfully control the inverted pendulum systems [6,7]. Fuzzy logic and artificial neural network controllers have been used to obtain more stable control in systems where disturbing input is present [9-12]. In order to achieve better results from classical PID and LQR type controllers, many studies that used some optimization methods such as bees algorithm [13, 14], genetic algorithm [15-17] and particle swarm optimization [17, 18] have been carried out so that the control parameters can be adjusted appropriately. In this study, optimization of the Linear Quadratic Regulator (LQR) parameters by using Particle Swarm Optimization (PSO) method for balance and position control of the inverse pendulum system is discussed. Equations of motion of the system are obtained by using the Lagrange formulation. These equations are linearized by using Taylor Series expansion and state-space model of the linearized system is obtained. LQR controller design was realized by using the state space model. An optimization method based on the PSO algorithm was presented to determine the parameters of the designed LQR controller. Simulation studies are performed in MATLAB/Simulink environment and the results are presented in order to examine the effectiveness of the LQR controller optimized by PSO algorithm.

II. Mathematical Model of Inverted Pendulum

Two degrees of freedom inverted pendulum (IP) system is shown in Fig. 1. The system is consisted of a cart and a pendulum connected to each other with rotary joint. The force F is applied to the car to keep the pendulum in a vertical position. A friction force acts on the car in the reverse direction of movement. The positions of the cart and the inverted pendulum are controlled only by the horizontal force F applied to the car. This system is a good example of an underactuated systems, since two different system variables are tried to be controlled by a single input.

Lagrange formulation was used to obtain mathematical equations of the system. Lagrangian of the system is,

$$L = T - V \tag{1}$$

whereT and V are the sum of the kinetical and potential energies of the cart and pendulum respectively. Lagrange formulation is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q_i \tag{2}$$

where q_i is the generalized coordinates and Q_i is the generalized forces of the system.

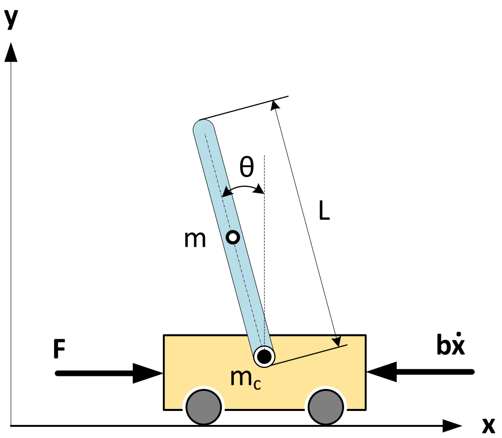

Fig. 1. Inverted pendulum model

For this problem, generalized force is F and generalized coordinates are,

$$q(t)^T = [x(t) \quad \theta(t)] \tag{3}$$

where x(t) is the position of the cart in the horizontal direction and θ(t) is the angle of the pendulum relative to the vertical direction.

If the kinetic and potential energies of the system are written in Eq. 1, Lagrangian of the system can be obtained as below.

$$L = 0.5(m_c + m_p)\dot{x}^2 + 0.3m_pL^2\dot{\theta}^2 + 0.5m_pL\dot{x}\dot{\theta}cos\theta - 0.5m_pgLcos\theta \tag{4}$$

If the Lagrangian of the system is written in the Eq. 2, equations of motion can be obtained as below.

$$\begin{aligned}(m_c + m_p)\ddot{x} + 0.5m_pLcos\theta\ddot{\theta} - 0.5m_pLsin\theta\dot{\theta}^2 &= F - b\dot{x}\\ 0.5m_pLcos(\theta)\ddot{x} + 0.6m_pL^2\ddot{\theta} - 0.5m_pLgsin(\theta) &= 0\end{aligned} \tag{5}$$

The equilibrium point of the system is $[x \quad \theta \quad \dot{x}\dot{\theta}] = [0 \;\; 0 \;\; 0 \;\; 0]$. It can be assumed that the system makes small oscillations around the equilibrium point. So, it can be linearized around the equilibrium point by using Taylor series expansion. State-Space model of the linearized model is obtained as below.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{6}$$

States (x) and outputs (y) of the system;

$$x^T = [x_1 x_2 x_3 x_4] = [x \quad \theta \quad \dot{x}\dot{\theta}]$$
$$y^T = [x_1 x_2] \tag{7}$$

TABLE 1. Parameters of the Inverted Pendulum System

Parameter	Definition
m_c=0.5 kg	Mass of the cart
m_p=0.2 kg	Mass of the pendulum
L=0.3 m	Length of the pendulum
I=0.006	Moments of inertia of the
g=9.81	Gravity acceleration
b=0.1	Coefficient of friction for cart

The state and output matrices of the system are calculated as follows by using the parameters given in Table 1.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.673 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.18 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1.818 \\ 0 \\ 4.545 \end{bmatrix} \tag{8}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9}$$

III. Adjustment of LQR Controller Parameters with Particle Swarm Optimization

The Linear Quadratic Regulator (LQR) is a control method that uses performance index and state variables to calculate the optimal control input. The J performance index including state errors and system input is given in Eq. 10. The LQR control method is based on minimizing the J performance index using state error and system input. In the LQR controller design, the system input is calculated to reduce the performance index J by using Q and R diagonal matrices given in Eq. 11.

In LQR control system input is expressed as $u = K.(ref - x)$ for $\dot{x} = Ax + Bu$. LQR gain is expressed as $K = R^{-1}B^T P$. In this equation, P is a symmetrical matrix obtained from Ricatti Equation given in Eq. 12 [19].

$$J = \int_0^\infty \left(q_{ref} - q(t)\right)^T Q \left(q_{ref} - q(t)\right) + u(t)^T R u(t)\, dt \tag{10}$$

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & q_n \end{bmatrix} R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & r_m \end{bmatrix} \tag{11}$$

$$A = A^T P + Q - PBR^1 B^T P = 0 \tag{12}$$

Determination of the LQR parameters is generally achieved by trial and error method. In addition to being extremely laborious, it is almost impossible to find the best parameters with this method. For this reason, many optimization methods such as genetic algorithm, the bees algorithm, PSO, etc. are used to determine optimum LQR parameters [13-18].

PSO algorithm is a population-based heuristic optimization method developed by Kennedy and Eberhart [20]. It has been inspired by the social behavior of birds or fish. Compared to other optimization methods, PSO has very few parameters and is easy to implement. In this algorithm, potential solutions called particles navigate the problem space by following the best available solutions. PSO basically relies on the approximation of the position of the individuals in the swarm to the individual with the best position of the swarm. The initial population is represented by a group of particles. According to the objective function, the fitness values of each randomly generated particle are calculated and p_{best}

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value is determined for each particle. The p_{best} value is the best fitness value a particle has ever found. The value of g_{best} is the best fitness value ever obtained by any particle in the population, and this value is the global best value for the population. According to the calculated p_{best} and g_{best} values, the velocities of the particles are updated according to Eq. 13 and their positions are updated according to Eq. 14.

$$v_{ij}^{k+1} = w.v_{ij}^{k} + c_1 .r_1 .(pbest_{ij}^{k} - x_{ij}^{k}) + c_2 .r_2 .(gbest_{j}^{k} - x_{ij}^{k}) \quad (13)$$

$$x_{ij}^{k+1} = x_{ij}^{k} + v_{ij}^{k+1} \quad (14)$$

where c_1 and c_2 are learning factors and guide the movement of the particle according to its own experience and the experience of the other particles in the swarm, respectively. r_1 and r_2 are the random values at[0,1] interval.w is the inertia weight and the small inertia weight enables the local search while the large inertia weight allows the global search. After the update, the fitness values of all particles in the new population are recalculated. These operations are repeated for the number of iterations given as input to the algorithm.

LQR control scheme of the inverse pendulum system is given in Fig 2.Inputs of the system are the desired positions of the car (x_{ref}) and pendulum (θ_{ref}) and outputs are the actual positions and velocitiesof the car (x, $\dot{x}$)andpendulum θ, $\dot{\theta}$. The force is produced by the LQR controller to move the cart to desired reference position and keep the system stabilized.

In this study, the parameters of Q and R matrices of pre-designed LQR controller are optimized by using PSO algorithm.The values to minimize the objective function of the system have been investigated.The objective function used for the optimization of the LQR controller is given in Eq. 15. Objective Function includes the parameters of peak time (t_p), rise time (t_r), settling time (t_s), steady state error (x_{sse}) and maximum peak(x_{max}) which are obtained from the time response of the system. The selected optimization intervals are given in Table 2.

$$Je = (x_{pt} + x_{rt} + x_{st} + x_{max} + x_{sse}) + (\theta_{norm} + \theta_{ts} + \theta_{tp} + \theta_{max} + \theta_{ess}) \quad (15)$$

TABLE 2. PARAMETERS OF THE INVERTED PENDULUM SYSTEM

	Q_1	Q_2	Q_3	Q_4	R
Min	0	0	0	0	0
Max	100	10	500	10	2

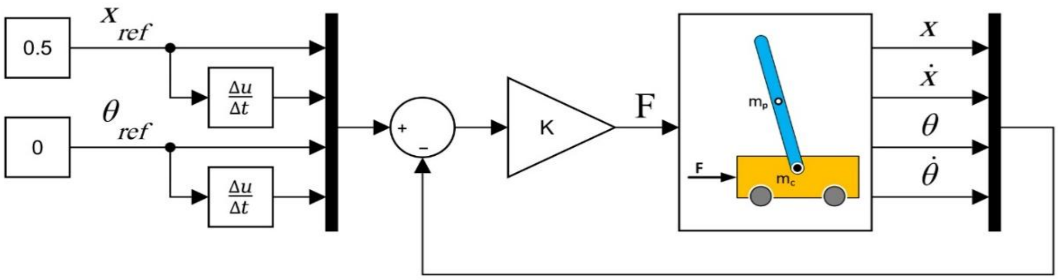

Fig. 2.LQR control scheme for inverted pendulum system

IV. SIMULATIONS AND RESULTS

In simulation studies, the optimization of the LQR controller designed for the control of the inverse pendulum system was performed using the PSO algorithm.The parameter values determined for the optimization are given in Table 3. The weight

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matrices (Q and R) and the gain matrix K calculated for the LQR controller after optimization are given in equations 16, 17 and 18 respectively.

TABLE 2. OPTIMIZATIONPARAMETERS

Parameter	Definition
c1, c2=2	Learning factors
IN=50	Iteration number
PN=30	Population number
Vmax=5	Maximum velocity of particles
Vmin=-5	Minimum velocity of particles
w=0.8	Inertia weight

$$Q = \begin{bmatrix} 64.059 & 0 & 0 & 0 \\ 0 & 0.068 & 0 & 0 \\ 0 & 0 & 265.428 & 0 \\ 0 & 0 & 0 & 1.713 \end{bmatrix} \quad (16)$$

$$R = 0.3883 \quad (17)$$

$$K = [-12.843 \quad -11.315 \quad 54.775 \quad 9.613] \quad (18)$$

Time-dependent changes of the cart position, pendulum angle and the control force are given in Fig. 3-5 respectively.Fig. 3 shows the variation of the cart position with time.As shown in Fig. 3, designed LQR controller was enable the cart to go to the desired position of 0.5 meters in about 3.5 seconds.Fig. 4 shows the variation of the pendulum angle with time.As shown in Fig. 4, pendulum angle was deviated approximately 7^{o}from the equilibrium after the reference input was applied to system, but the designed LQR controller successfully returned the pendulum to the equilibrium position and stabilized it.It can be seenfrom Fig. 5 that the control force applied during the control process was below 2 N.

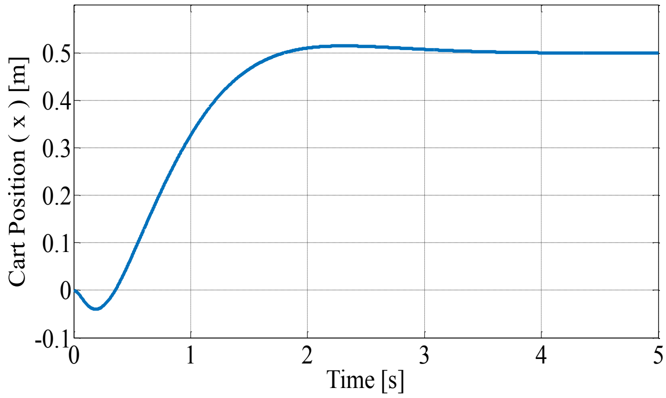

Fig. 3. Variation of the cart position with time

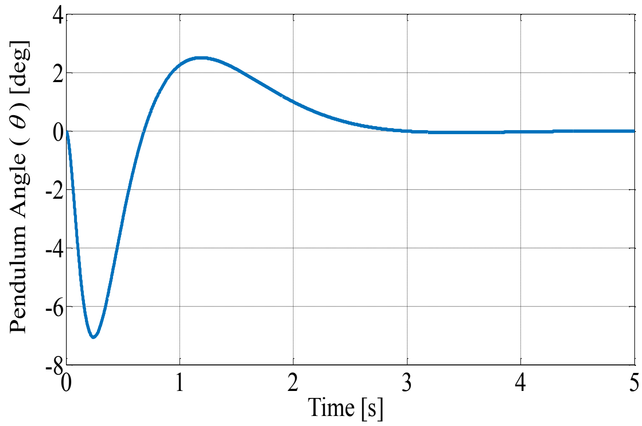

Fig. 4. Variation of the pendulum angle with time

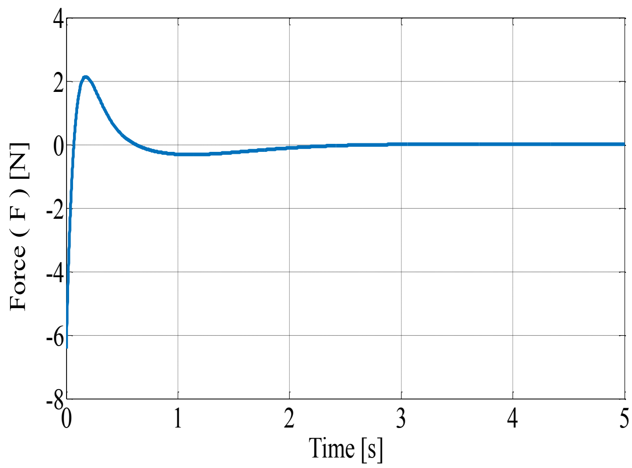

Fig. 5. Time-dependent change of control force

CONCLUSIONS

In this study, PSO based optimization of a LQR controller designed for position and stabilization control of an inverse pendulum system is presented.Equations of motion of the inverted

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pendulum system were obtained by using Lagrange formulation and linearized around the equilibrium point by Taylor series expansion. A LQR controller was designed by using state-space model of the system. The parameters (Q and R matrices) of the designed LQR controller are optimized by the PSO algorithm to obtain the gain matrix K of the LQR controller. An objective function including the parameters of peak time (t_p), rise time (t_r), settling time (t_s), steady state error (x_{sse}) and maximum peak(x_{max}) which are obtained from the time response of the system was used optimization prosses. The performance of the optimized LQR controller has been investigated through simulation studies in MATLAB/Simulink environment.Simulation results shown that the optimized LQR controller successfully controls the position and stabilization of the inverse pendulum system.

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