

Mathematical Modeling of the Flow Variation of Natural Gas Extraction Wells Depending on the Diameter of the Mixing Pipe

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ABSTRACT: The exploitation of natural gas deposits also requires the transport of the extracted fluids through pipelines to their treatment facilities. Usually the extracted fluids contain, in addition to methane, ethane, liquid fractions of propane, butane, hexane and sometimes salt water associated with the deposit. Given that the delivery of natural gas for consumption must be carried out without water and oil liquids, the flow rates of the gas extraction wells are taken in two-phase mixing pipelines and transported to a single treatment facility. The article presents the impact of the diameter of the mixing pipes on the gas flow.

KEYWORDS: natural gas, pipeline, biphasic, oil fluids.

I. INTRODUCTION

The transport of gases extracted from exploitation wells is carried out in mixed conditions, namely methane, ethane, propane, butane, hexane and water associated with the deposit. During the flow of two phases (gases and liquids) through pipelines there is a change in pressure variation (a pressure drop) between the two transported phases and therefore the possibility of product separation. This separation is manifested by the deposition of the liquid phase at the bottom of the pipe.

The flow regimes associated with two-phase motion through horizontal pipes have been approximated (Figure 1) with flow in the form of dispersed bubbles (very small gas-liquid ratios settling at the bottom of the pipe and gas bubbles rising to the top), elongated bubbles (when the gas-liquid ratio increases and bubbles become larger forming plugs), stratified flow (when gases and liquids form separate layers, in this case gas plugs joining at the top of the pipe), wave flow (when there high gas ratios in the biphasic mixture), plug flow (at high gas ratios in the biphasic mixture a gas separation phenomenon occurs in plugs with lengths of tens of meters) and ring fog (at extreme gas-liquid ratios from the transported fluid, the liquid becomes dispersed in the gas forming a fog-like gas-liquid atmosphere). Usually, the calculation of pressure drops of two-phase fluids transported through pipelines requires the evaluation of phase changes, which are due to changes in pressure and temperature in the pipeline, as well as the evaluation by empirical means of liquid retention at the bottom of the pipeline.

It is also important to evaluate the energy transfer between the phases.

II. NUMERICAL MODELING OF FLUID FLOW THROUGH MIXING PIPES

One of the most useful relations for calculating the pressure loss between phases in two-phase flow through pipes is a formula derived from the general equation of isothermal flow. It should be noted that this relationship, determined experimentally, is only useful when the pressure drop is less than 10% of the pipe inlet pressure [1]:

$$\Delta p = 62,561 \frac{Q_{am}^2 \cdot L \cdot \lambda}{\rho_{am} \cdot d^5}, \text{ (kPa)} \quad (1)$$

In relation 1, λ represents the resistance coefficient of the pipe of length L (m) and internal diameter d (mm), through which flows a two-phase fluid flow Q_{am} (kg/h), with a density ρ_{am} (kg/m³).

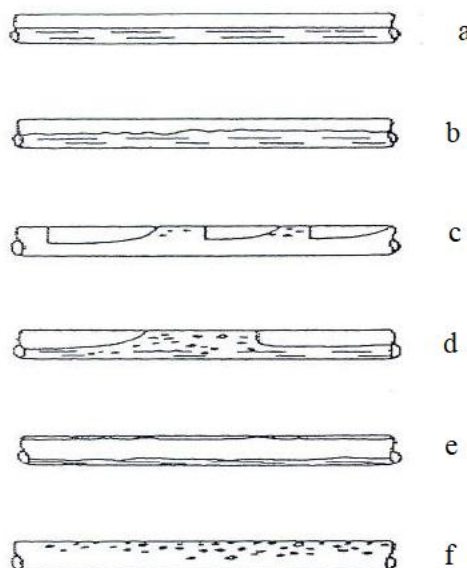


Fig. 1. Two-phase movement of fluids in mixing pipes

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a. smooth layered, b. layered in waves, c. elongated bubbles, d. plug, e. ring fog, f. dispersed bubbles [1].

As can be seen in relation 1, an average value of the mixture density and the fact that this pipe is perfectly horizontal were taken into account.

Taking into account the assumptions mentioned above, the mass flow rate of the mixture can be determined with the experimental equation [2,3,4]:

$$Q_{am} = (0,00121 \cdot Q_g \cdot \delta + 0,9997 \cdot Q_l \cdot \gamma) / 100, \text{ (kg/h)} \quad (2)$$

In relation 2 it is approximated that the gas flow rate Q_g under standard conditions (m^3/day) represents only 0.00121% of the mixture and the liquid flow rate Q_l (m^3/h) represents 0.997% of the mixture. For the accuracy of the calculation, the relative densities of gases compared to air δ and of liquids compared to water γ were introduced in the relationship.

And the mixture density ρ_{am} was determined experimentally according to relation 3:

$$\rho_{am} = \frac{28,814 \cdot \gamma \cdot \rho + 34,81 \cdot R \cdot \delta \cdot p}{28,82 \cdot p + 10 \cdot R_{St} \cdot T \cdot Z} \text{ (kg/m}^3\text{)} \quad (3)$$

Where, in addition to the relative densities of the analyzed gases and liquids, the value of the gas-liquid ration (Nm^3/m^3) was also used.

For the correction of the density value of the mixture at the calculation temperature T (K) and pressure p (kPa), the deviation factor Z of gases was introduced.

But the errors are quite large in the use of these relationships and that is why several thermodynamic models are used to calculate the pressure losses as well as to determine the variation of the mixture density and the mixture flow rate depending on the mixing ratio of these two phases.

A useful calculation method is the one formulated by Beggs and Brill [5,6,7], which uses the pressure equation by writing the energy balance of a kg of fluid flowing between two points.

$$Vdp + gdh + v_m dv_m + dF = 0 \quad (4)$$

or

$$\frac{dp}{\rho_m} + gdh + v_m dv_m + dF = 0 \quad (5)$$

The energy balance written for the distance between the two considered points dh (m) of a two-phase fluid with a specific volume V (m^3/kg), density ρ_m (kg/m^3) and with a mixing speed v_m ($v_m = (q_l + q_g)/A$) (m/s) has a frictional energy loss dF . As we note in relation 5, we considered the average velocity to be given by the flow of a two-phase fluid consisting of a liquid (flowing with the flow rate q_l (m^3/s)) and a gas (flowing with the flow rate q_g (Nm^3/s)), through a flow area A (m^2).

Considering that in reality, the pipes are not installed horizontally, the vertical distance dh (m) was corrected according to the installation angle of the pipe α (relative to the horizontal) and the length of the analyzed pipe dz (m).

$$dh = dz \sin \alpha \quad (6)$$

The angle α has the value 0° when the flow occurs through horizontally mounted pipes and varies between 0° and $+90^\circ$ for flow through pipes inclined from the bottom up and from 0° to -90° for flow through pipes inclined from above in down.

Equation 5 can be written as the pressure loss in the pipe length as:

$$\frac{dp}{dz} = - \left(\rho_m g \sin \alpha + \frac{\rho_m v_m dv_m}{dz} + \rho_m \frac{dF}{dz} \right) \quad (7)$$

$$- \frac{dp}{dz} = \left(\frac{dp}{dz} \right)_{static} + \left(\frac{dp}{dz} \right)_{acc} + \left(\frac{dp}{dz} \right)_{fr} \quad (8)$$

So the static gradient becomes:

$$\left(\frac{dp}{dz} \right)_{static} = g \rho_m \sin \alpha \quad (9)$$

We can also define the density of the mixture ρ_m depending on the density of the liquid ρ_l (kg/m^3) and the gas ρ_g (kg/m^3).

Relation 3 can be reduced to the equation

$$\rho_m = \rho_l \varepsilon_l + \rho_g (1 - \varepsilon_l) \quad (10)$$

In which we used the liquid fraction as the ratio of the volume of liquid in an element relative to the volume of the element ε_l

The value of the static gradient results as:

$$\left(\frac{dp}{dz} \right)_{static} = g [\rho_l \varepsilon_l + \rho_g (1 - \varepsilon_l)] \sin \alpha \quad (11)$$

To determine the value of the pressure gradient due to the acceleration of the fluid mass, the following relationship is used:

$$v_m = v_{sl} + v_{sg} = \frac{G_l}{\rho_l} + \frac{G_g}{\rho_g} \quad (12)$$

where:

- v_{sl} and v_{sg} – the superficial velocities of the liquid and gases resulting by dividing the volumetric flow rates by the sections occupied by liquid and gas, respectively;

- G_l – the ratio between the liquid mass flow rate and the total pipe section, $\text{kg}/\text{s}\cdot\text{m}^2$, and

- G_g – the ratio between the gas mass flow rate and the total pipe section, $\text{kg}/\text{s}\cdot\text{m}^2$.

Therefore, the pressure gradient calculated during the movement of fluids through pipes can be written:

$$\left(\frac{dp}{dz} \right)_{acc} = \rho_m v_m \frac{dv_m}{dz} = \rho_m v_m \left[\frac{d}{dz} \left(\frac{G_l}{\rho_l} \right) + \frac{d}{dz} \left(\frac{G_g}{\rho_g} \right) \right] \quad (13)$$

It is assumed that $\left[\frac{d}{dz} \left(\frac{G_l}{\rho_l} \right) \right] = 0$ because of the difference between gas and liquid compressibility.

$$\left(\frac{dp}{dz} \right)_{acc} = \rho_m v_m \left[\frac{d}{dz} \left(\frac{G_g}{\rho_g} \right) \right] = \rho_m v_m \left[\frac{\rho_g \frac{dG_g}{dz} - G_g \frac{d\rho_g}{dz}}{\rho_g^2} \right] = \rho_m v_m \left[\frac{dG_g}{dz} - \frac{G_g}{\rho_g^2} \frac{d\rho_g}{dz} \right] \quad (14)$$

It can also be admitted that the change in gas mass flow, through the exit or entry of part of the gas into the solution, is extremely small compared to the change in gas density:

$$\frac{d}{dz} \left(\frac{G_g}{\rho_g} \right) \ll \frac{G_g}{\rho_g^2} \frac{d\rho_g}{dz}, \quad (15)$$

from which it follows:

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$$\left(\frac{dp}{dz}\right)_{acc} = -\rho_m v_m \frac{G_g}{\rho_g^2} \frac{d\rho_g}{dz} \quad (16)$$

On the other hand, for 1 kg of gas, one can write:

$$\rho_g = \frac{pM_g}{zRT} \quad (17)$$

$$\frac{d\rho_g}{dz} = \frac{d}{dz} \left(\frac{pM_g}{zRT} \right) = \frac{M_g}{zRT} \frac{dp}{dz} + \frac{p}{zRT} \frac{dM_g}{dz} - \frac{pM_g}{z^2 RT} \frac{dz}{dz} - \frac{pM_g}{zRT^2} \frac{dT}{dz} \quad (18)$$

We can admit that the sum of the last three terms on the right side of relation (18) is negligible compared to the first term, so:

$$\frac{d}{dz} (\rho_g) = \frac{M_g}{zRT} \frac{dp}{dz} \quad (19)$$

Taking into account relation (18), relation (19) becomes:

$$\frac{d}{dz} (\rho_g) = \frac{\rho_g}{p} \frac{dp}{dz}, \quad (20)$$

where p is the pressure at the point where the pressure gradient is calculated.

Substituting relation (20) into equation (14) results:

$$\left(\frac{dp}{dz}\right)_{acc} = -\rho_m v_m \left[\frac{G_g}{\rho_g^2} \frac{\rho_g}{p} \frac{dp}{dz} \right] = -\rho_m v_m \frac{v_{sg}}{p} \frac{dp}{dz} \quad (21)$$

The pressure gradient due to friction is given by the relation:

$$\left(\frac{dp}{dz}\right)_{fr} = \frac{f_{bifazic} \rho_{fa} v_m^2}{2d} = \frac{f_{bifazic} G_m v_m}{2d} \quad (22)$$

where $G_m = G_l + G_g$ iar

$$\rho_{fa} = \rho_l \frac{q_l}{q_l + q_g} + \rho_g \left(1 - \frac{q_l}{q_l + q_g} \right).$$

As can be seen, ρ_{fa} represents a weighted density of the biphasic mixture but without taking into account the phenomenon of gas sliding through the liquid.

This sliding of gases through liquid usually corresponds to an identical velocity for both gas and liquid.

In biphasic flow, ρ_{fa} differs depending on the specific weight of the mixture ρ_m calculated, taking into account slip.

By substituting the above relationships in equation (8) we obtain successively

$$-\frac{dp}{dz} = g[\rho_l \varepsilon_l + \rho_g(1 - \varepsilon_l)] \sin \alpha + \frac{f_{bifazic} G_m v_m}{2d} - \frac{[\rho_l \varepsilon_l + \rho_g(1 - \varepsilon_l)] v_m v_{sg}}{p} \frac{dp}{dz} \quad (23)$$

$$-\frac{dp}{dz} = \frac{g[\rho_l \varepsilon_l + \rho_g(1 - \varepsilon_l)] \sin \alpha + \frac{f_{bifazic} G_m v_m}{2d}}{1 - [\rho_l \varepsilon_l + \rho_g(1 - \varepsilon_l)] v_m v_{sg} / p} \quad (24)$$

Equation (24) contains two unknowns: ε_l required for the calculation of the density of the mixture at a given point and $f_{bifazic}$ required for the calculation of frictional pressure losses.

Enter the expressions

$$N_{Fr} = \frac{v_m^2}{gd} \quad (25)$$

$$\lambda = \frac{q_l}{q_g + q_l} \quad (26)$$

Where:

$$L_1 = \exp(-4,62 - 3,754 \cdot \ln \lambda - 0,481 \cdot \ln^2 \lambda - 0,0207 \cdot \ln^3 \lambda) \quad (27)$$

$$L_2 = \exp(1,061 - 4,602 \cdot \ln \lambda - 1,609 \cdot \ln^2 \lambda - 0,179 \cdot \ln^3 \lambda + 0,635 \cdot 10^{-3} \cdot \ln^5 \lambda) \quad (28)$$

Depending on the values of L_1 and L_2 , the biphasic flow of fluids through pipes will be of the gravity segregation type ($N_{Fr} < L_1$), uniformly distributed flow ($N_{Fr} > L_1$ and $N_{Fr} > L_2$) or intermittent flow ($L_1 < N_{Fr} < L_2$).

After determining the values of $\varepsilon_l(0)$ and C, the value of the liquid fraction is calculated for a certain angle.

Result:

$$\varepsilon_l(\alpha) = \varepsilon_l(0) \left[1 + C \left(\sin 1,8 \cdot \alpha - \frac{1}{3} \sin^3 1,8 \cdot \alpha \right) \right] \quad (29)$$

From this correlation we determine the friction factor of the fluids against the wall of the pipes through which they flow:

$$\frac{f_{bifazic}}{f_{fa}} = f \left\{ \frac{\lambda}{[\varepsilon_l(\alpha)]^2} \right\} = eS \quad (30)$$

Where f_{fa} is the sliding phenomenon and can be obtained as a function of the Reynolds number given by:

$$\left(N_{Re_{fa}} = \frac{[\rho_l \lambda + \rho_g(1 - \lambda)] \cdot v_m \cdot d}{\mu_l \lambda + \mu_g(1 - \lambda)} \right) \quad (31)$$

and

$$S = \frac{\ln(y)}{-0,0523 + 3,182 \ln(y) - 0,872 [\ln(y)]^2 + 0,01853 [\ln(y)]^4}, \quad (32)$$

Where

$$y = \lambda / [\varepsilon_l(\alpha)]^2.$$

For the interval $1 < y < 1,2$ the function S is calculated with the relation:

$$S = \ln(2,2y - 1,2) \quad (33)$$

To calculate the pressure gradient at a point, proceed as follows:

- calculate $\rho_l, \rho_g, v_{sl}, v_{sg}, v_m, G_m, \lambda, N_{Fr}, (N_{Re})_{fa}$ and N_{lv} at the pressure and temperature of the respective point,
- calculate L_1 and L_2 and determine the flow regime,
- $\varepsilon_l(0)$ is calculated,
- C is calculated,
- the ratio $\varepsilon_l(\alpha) / \varepsilon_l(0)$,
- $\varepsilon_l(\alpha)$ and ρ_m are determined,
- the $f_{bifazic} / f_{fa}$ ratio is calculated,
- the value of f_{fa} is determined from the Moody Diagram (fig. 2),
- dp/dz is determined.

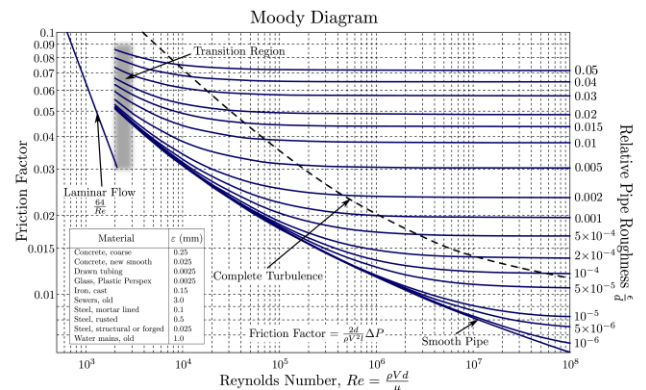


Fig. 2. Moody diagram [1]

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Table 1. Calculation relation for C

Flow regimes through horizontal pipes	Liquid fraction for flow through horizontal pipes	C +
Gravitational segregation	$\varepsilon_l(0) = \frac{0,98\lambda^{0,4868}}{N_{Fr}^{0,0868}}$	$C+ = (1 - \lambda) \ln \frac{0,011N_{lv}^{3,359}}{\lambda^{3,768}N_{Fr}^{1,614}}$
Flashing	$\varepsilon_l(0) = \frac{0,845\lambda^{0,5351}}{N_{Fr}^{0,0173}}$	$C+ = (1 - \lambda) \ln \frac{2,96\lambda^{0,365}N_{Fr}^{0,0978}}{N_{lv}^{10,4473}}$

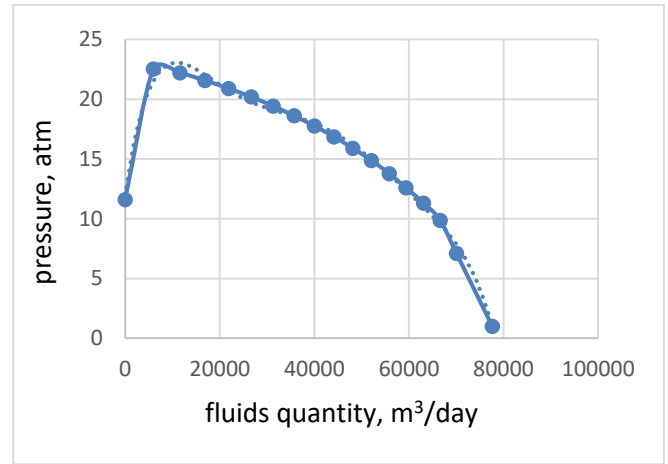


Fig.3. The flow rate of the well depending on the pressure of the productive layer

Flow regimes through horizontal pipes	Liquid fraction for flow through horizontal pipes	C -
Gravitational segregation	$\varepsilon_l(0) = \frac{0,98\lambda^{0,4868}}{N_{Fr}^{0,0868}}$	$C- = (1 - \lambda) \ln \frac{4,7N_{lv}^{0,1244}}{\lambda^{0,3692}N_{Fr}^{0,5056}}$
Flashing	$\varepsilon_l(0) = \frac{0,845\lambda^{0,5351}}{N_{Fr}^{0,0173}}$	$C- = (1 - \lambda) \ln \frac{4,7N_{lv}^{0,1244}}{\lambda^{0,3692}N_{Fr}^{0,5056}}$

Table 2. well gas and oil properties

Name of properties	Quantity	Measurement unit
Gas flow (measured)	28150	m³N/zi
Water flow	2.2	m³/zi
Condens flow	3.5	m³/zi
Reservoir pressure	27.1	bar
Reservoir permeability	746	mD
Reservoir thickness	73	m
Perforation interval	6	m
Reservoir temperature	48	C
Reservoir area	488	ha
Water density	1070	kg/m3
Condens density	780	kg/m3
CO2 content	0.32	%
H2S content	0	%
N2 content	0.68	%
Perforation density	11	SPM
Perforation diameter	43	mm
kc / kf ratio	0.6	
Damaged zone permeability	41	mD
Damaged zone radius	850	mm
Flow line length	3500	m
Flow line diameter	52.5	mm
Separator pressure	15	bar

III. MATHEMATIC MODELLING OF FLUIDS FLOW IN THE MIXING PIPES

Taking a probe similar to the one for which the simulation must be carried out (the probe with the data from table 2), we were able to write the equations for modeling the fluid flows as a function of the reservoir pressure (fig. 3, 4, 5, 6 and table 3).

The analysis in the gas field of the flow of petroleum fluids extracted from the well as a function of the fluid pressure, showed a variation of the form given in figure 3.

Figures 4, 5, and 6 show the flow rates of petroleum fluid through 2-inch, 3-inch, and 4-inch pipes.

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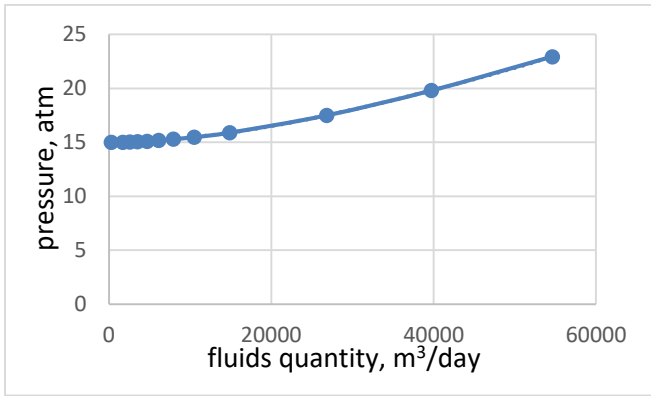


Fig. 4. Fluid flow rate through 2-inch pipe as a function of reservoir pressure

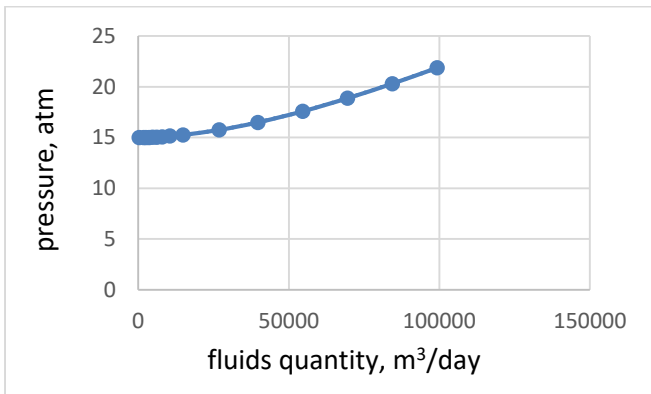


Fig. 5. Fluid flow rate through 3-inch pipe as a function of reservoir pressure

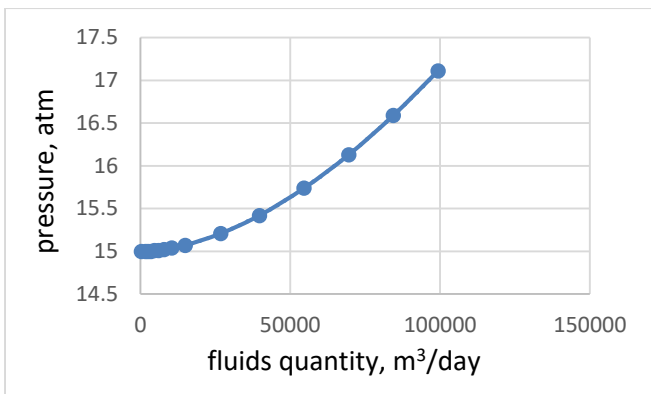


Fig. 6. Fluid flow rate through 4-inch pipe as a function of reservoir pressure

Table 3. Equations for modeling petroleum fluid flows x, (m³/day) as a function of reservoir pressure y (atm)

Flow description	equation	R ²
inflow	$y = -7E-27x^6 + 2E-21x^5 - 2E-16x^4 + 1E-11x^3 - 2E-07x^2 + 0,0028x + 11,891$	0,994 2
Pipeline 2 inch	$y = 3E-28x^6 - 4E-23x^5 + 3E-18x^4 - 1E-13x^3 + 5E-09x^2 + 4E-06x + 14,999$	1

Pipeline 3 inch	$y = 3E-30x^6 - 1E-24x^5 + 1E-19x^4 - 1E-14x^3 + 1E-09x^2 + 3E-06x + 14,996$	1
Pipeline 4 inch	$y = 2E-30x^6 - 6E-25x^5 + 8E-20x^4 - 6E-15x^3 + 4E-10x^2 - 6E-08x + 14,999$	1

CONCLUSIONS

The analysis of the flow of two-phase fluids through the pipelines, combined with the flow of two-phase fluids in the well, could lead to the selection of an optimal flow rate for all three pipelines.

This is given in figure 7 and as seen the highest value of 49367 m³/day at a pressure of 15.9 atm can be obtained by using a 4 inch pipe.

But there is a danger that at this flow rate the oil product will segregate and then the best solution is to use a 3 inch pipe with a flow rate of 44319 m³/day and a pressure of 16.7 atm.

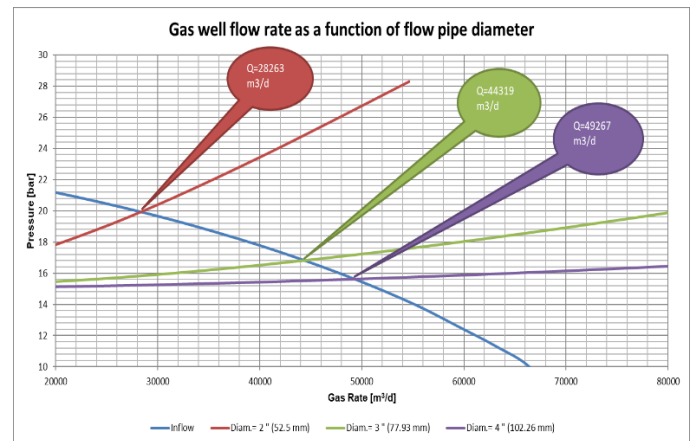


Fig.7. Optimal pressure and pipe diameter to transport fluids

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