

Modelling and Sensitivity Analysis of Influencing Parameters in Displacement of Dynamic Bodies

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ABSTRACT: The mathematical modelling in relation to the Six-degree freedom system of train suspension is developed and simulated for their dynamic characteristics. The important challenge in the suspension system is vertical displacement obtained from the vehicle body. To reduce vertical displacement, an analysis of the model is done by variation of parameters such as stiffness of spring and damping coefficient. The model has been created by deriving the equations of a system using Newton's law. The developed model has the potential to analyse the dynamic characteristics of the suspension system for both displacement of the vehicle body and displacement of the wheel. The outcome of this research revealed that Secondary spring stiffness is the most dominant factor to influence the displacement of the vehicle body; Primary damping coefficient is the most dominant factor to influence displacement of the wheel.

KEYWORDS: Suspension, Train Bogie, Damping, spring, and Sensitivity analysis.

1. INTRODUCTION

In train bogie, suspension system is important in reducing as much vibration and shocks occurring in the system. It providing stability of vehicle body and passenger comfort. Suspension framework configuration is a difficult task for the vehicle designers considering different control parameters, complex goals. For vehicles, it is continually testing to keep up at the same time an elevated expectation of ride comfort, vehicle taking care of under every single driving condition. The target of this paper is to build up a MATLAB/SIMULINK model of train suspension to analyse the dynamic behaviour. Just as the detailed mathematical modelling displaying with step by step arrangement of equations are created and validation of Simulink model is to be done elaborately on this paper. Most of the research article have been directed to Modelling and analysis of control the vibration vehicle body. A. Mitra [1], has developed a full car mathematical model by deriving the equation of motion from free body diagram. They also validated for various road profile on a Simulink software and analytical solution is done on this paper. P.M. Jawahar [2], derived the equations of motion of the rail vehicle by building a nonlinear mathematical model. They studied the railway vehicle's complex response. By Adam's process, the equation of motion is transformed into a type of numerical integration. A computer program is set up to analyse the complex response of a rail vehicle system. The results show that the dynamic characteristics are improved favourably by the unconventional system relative to the conventional system. The FIAT (Fabbrica Italiana de Automobile Torino) [3] bogie

is a selection of EUROFIMA design. The FIAT bogie has a place with the two-axle, with an essential Primary and Secondary suspension system. A control arm gives axle direction. This is a two-stage suspension bogie. Secondary suspension holds the train body and primary suspension holds the bogie frame over the axle body. The following and slowing down power from hub to bogie is moved through expressed control arm arrangement of primary suspension. The physical model of FIAT bogie is shown in fig. 1. The FIAT bogie [4] outline is comprised of two longitudinal segments associated by two cross beams. Primary suspension framework comprises of settled internal and external springs, dampers, and top elastic cushion. A verbalized control arm fitted with twin-layer versatile joints interfacing the hub bearing to the bogie frame. Secondary suspension system comprises of two spring packs which support the bogie outline. Each spring pack is made up by an interior and outer spring. An Anti-move bar fitted on the bogie outline understands a steady, diminished tendency coefficient during running. The bogie outline is connected to the reinforce pillar through two vertical dampers, a parallel damper, four wellbeing links and the foothold bars. The bogie outline is connected to the mentor body through two yaw dampers.

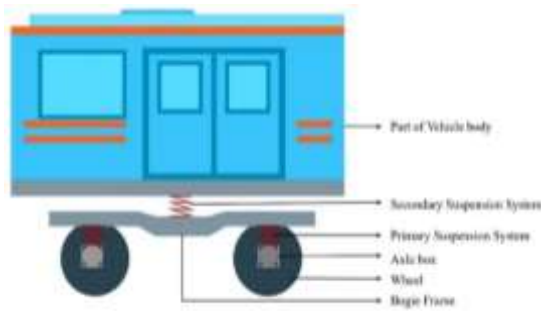


Figure 1. Physical Model of bogie

Mathematical modelling is widely used to study physical systems. The design wants to implement reasonable lateral stability for travelers, reasonable control surfaces and sufficient regulation of vehicle dynamics with the limit of making a small changes on suspension operating space. The key characteristic of all these designs is that they've been built on the principle of a physical system. The sprung mass and unsprung mass are modelled as rigid masses [5]. Simulations of these models can be done in MATLAB. It is therefore necessary to research a vehicle's responsiveness to random excitement. Using these designs, it can be helpful for an engineer to do mathematical equations initially. In this research Mathematical modelling, Simulink model, Sensitivity analysis and Pareto analysis are done. Finally, the findings of simulations performed in MATLAB are examined, accompanied by a conclusion.

2. METHODOLOGY

The dynamic modelling and simulation of the train suspension system was carried out using MATLAB-Simulink. The dynamic simulation approach was chosen because it studies the system's actions, compares and calculates the deviation from set points while compensating for the errors produced until the optimal performance of the system is obtained by optimization of the process parameters. Initial phase in the mathematical modelling is to represent the real system in form of free body diagram of the train body and its suspension structure [6]. Next phase is the formulation of governing equations as well as motion equations, preceded by the portrayal of the train suspension in the MATLAB-Simulink focused on the equations formulated. Final phase in this research work is optimization of process parameters based on sensitivity analysis and Pareto analysis.

2.1. Mathematical modelling of Suspension system

Mathematical modelling is a method for developing mathematical equations and describing a system. Here a mathematical model was developed and derived the equations of motion of a Train bogie Suspension system. For analysing Acceleration of Train Suspension system, vehicle individual wheelset and bogie are considered. From the physical model of Bogie we have obtained a free-body diagram of train suspension system.

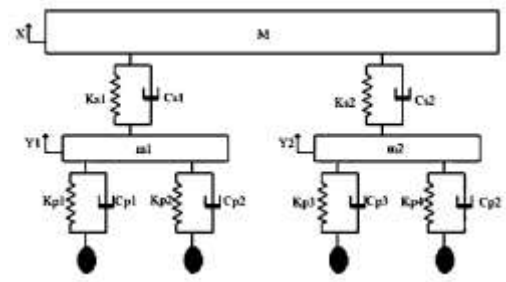


Figure 2. Mathematical model.

In mathematical modelling, the concept of a free-body diagram was used to obtain the differential equation of motion. The following three equations (2), (4) and (6) of motion are used to develop a SIMULINK model [7].

Equation for vehicle body

$$M\ddot{X} = -C_{s1}(\dot{X} - \dot{Y}_1) - K_{s1}(X - Y_1) - C_{s2}(\dot{X} - \dot{Y}_2) - K_{s2}(X - Y_2) \tag{1}$$

$$\ddot{X} = \frac{1}{M} \left(-C_{s1}(\dot{X} - \dot{Y}_1) - K_{s1}(X - Y_1) - C_{s2}(\dot{X} - \dot{Y}_2) - K_{s2}(X - Y_2) \right) \tag{2}$$

Equation for left wheel

$$M_1\ddot{Y}_1 = C_{s1}(\dot{X} - \dot{Y}_1) + K_{s1}(X - Y_1) - C_{p1}(\dot{Y}_1 - \dot{Z}) - K_{p2}(Y_1 - Z) - C_{p2}(\dot{Y}_1 - \dot{Z}) - K_{p2}(Y_1 - Z) \tag{3}$$

$$\ddot{Y}_1 = \frac{1}{M_1} \left(C_{s1}(\dot{X} - \dot{Y}_1) + K_{s1}(X - Y_1) - C_{p1}(\dot{Y}_1 - \dot{Z}) - K_{s2}(Y_1 - Z) - C_{p2}(\dot{Y}_1 - \dot{Z}) - K_{p2}(Y_1 - Z) \right) \tag{4}$$

For right wheel

$$M_2\ddot{Y}_2 = C_{s2}(\dot{X} - \dot{Y}_2) + K_{s2}(X - Y_2) - C_{p3}(\dot{Y}_2 - \dot{Z}) - K_{p3}(Y_2 - Z) - C_{p4}(\dot{Y}_2 - \dot{Z}) - K_{p4}(Y_2 - Z) \tag{5}$$

$$\ddot{Y}_2 = \frac{1}{M_2} \left(C_{s2}(\dot{X} - \dot{Y}_2) + K_{s2}(X - Y_2) - C_{p3}(\dot{Y}_2 - \dot{Z}) - K_{p3}(Y_2 - Z) - C_{p4}(\dot{Y}_2 - \dot{Z}) - K_{p4}(Y_2 - Z) \right) \tag{6}$$

2.2 Simulink model of train suspension system

The theoretical solution to the equations of motion could not be obtained. So development of the suspension system is accompanied by the creation of a model that optimizes the issue of seven degrees of freedom into a variation multisystem. A control issue is the analysis of a train suspension system's actions to ensure its reliability. Simulation model is therefore used for determining the acceleration response of Train body [8]. For development of Simulink model, solution of motion equation (2), (4) and (6) are considered. The train body is designed to have strong rail gripping capability, whereas the vertical displacement is held to have minimum. The Simulink Model is therefore optimized for both the displacement of the vehicle body and wheel displacement to evaluate the dynamic behaviour in time domain as shown in Figure 3. For the simulation analysis, the fixed parameters for the train suspension system

described in different research papers are taken and shown in Table 1.

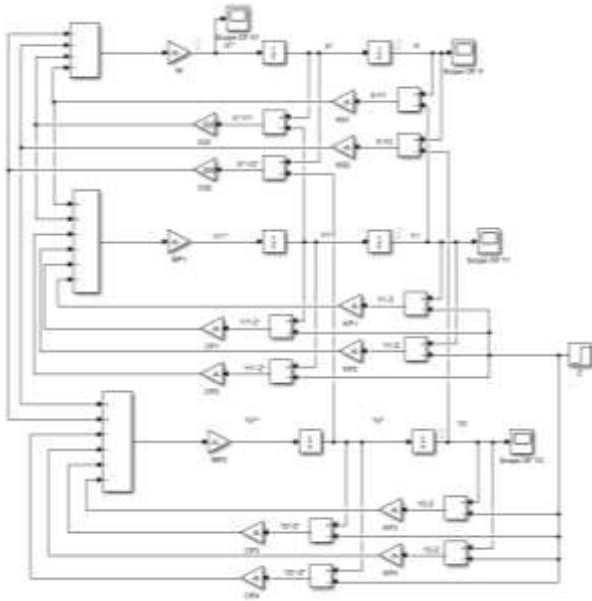


Figure 3. Simulink model

Table 1. Fixed parameters.

Mass of vehicle body (M)	354000 kg
Mass of Left wheel Suspension system (m1)	3100 kg
Mass of Right wheel Suspension system (m2)	3100 kg

2.3. Sensitivity Analysis

The sensitivity analysis, is used to determine how different factors affects the displacement of train suspension and we can determine the possibility of interactions between factors. Mostly, Taguchi method is used as optimization for sensitivity parametric analysis. This method is applicable when the design factors are qualitative and independent. It further gives a whole origination of interlinkage between the designs of components. Therefore, we can find the different issues identified to design and investigate the good design and performance. In simulation, the permutation and variation of spring stiffness and damping coefficient are analysed [9]. In this work, two levels and four factor of vehicle body parameters are selected to conduct the further simulation of train suspension that is shown in table 2.

Table 2. Parameters and levels values.

Parameters	Symbol	Level 1	Level 2
Primary spring stiffness	K_P	1150	755
Secondary spring stiffness	K_S	755	1150
Primary damping coefficient	C_P	1000	3000
Secondary damping coefficient	C_S	3000	1000

3. RESULTS AND DISCUSSION

Input data includes primary spring stiffness, primary damping coefficient, Secondary spring stiffness and secondary damping coefficient. In this stage equation of system derived from free body diagram are governed and assembled by utilizing a MATLAB software as Simulink model. By using this Simulink model (from appendix) output response of displacement of Vehicle body and displacement of wheel for train suspension is shown in table 3 by performing several runs in MATLAB software.

Table 3. Result on Maximum displacement of vehicle body and wheel of Train Suspension system.

Run	K_P (KN/m)	K_S (KN/m)	C_P (Ns/m)	C_S (Ns/m)	Maximum displacement	
					Vehicle body	Wheel
1	1150	755	1000	3000	1.983	1.420
2	1150	755	1000	1000	1.989	1.543
3	1150	1150	3000	3000	1.948	1.394
4	1150	1150	3000	1000	1.951	1.438
5	755	755	3000	3000	2.000	1.377
6	755	755	3000	1000	2.005	1.402
7	755	1150	1000	3000	1.924	1.565
8	755	1150	1000	1000	1.927	1.673

3.1. Mean effect plot for mean displacement

The main effects for mean displacement at each level vs the factors of Primary spring stiffness, Secondary spring stiffness, Primary damping coefficient, and Secondary damping coefficient values are plotted and shown in figure 4. The plot illustrates the effect of each level of variables based on the train suspension system. The level of the main contribution is chosen from the plot and the adjusted levels for the relevant element are the optimized.

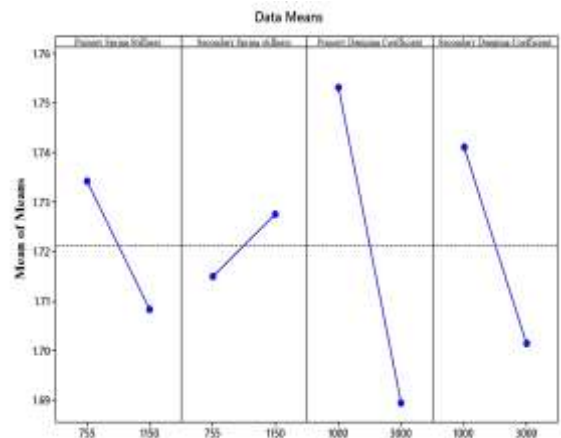


Figure 4. Main effect plot for mean displacement.

3.2 Optimal Solution

The optimal values of input parameters for train suspension system and corresponding response of displacement values

are shown in table 4. The optimal values are obtained by optimization of Table 3. Result on Maximum displacement of vehicle body and wheel of Train Suspension system values. The optimization are done in MINITAB software. By using optimal values shown in Table 4 the simulation run is conducted in MATLAB and results of displacement of vehicle body and wheel are shown in figure 5.

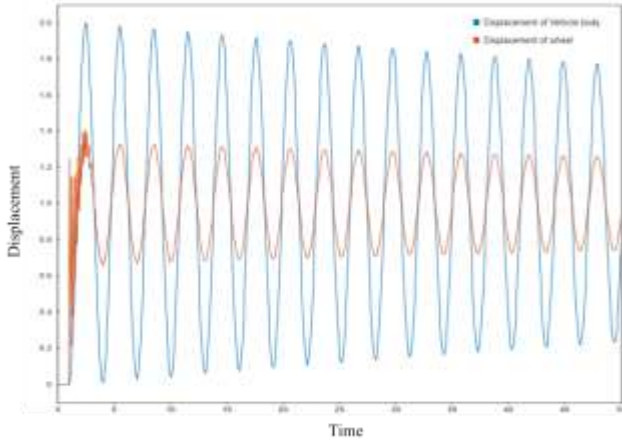


Figure 5. Results of Displacement of Vehicle body and Wheel

Table 4. Optimal Solution

Solution	K _P	K _S	C _P	C _S	Maximum displacement	
					Vehicle body	Wheel
1	1150 KN/m	1150 KN/m	3000 Ns/m	3000 Ns/m	1.94738	1.394

3.3. Pareto analysis

Pareto Analysis is a decision-making statistical tool used to choose a possible outcome that results from evaluation of the dominant factor. From the figure 6a, Pareto diagram for Standardized effect on displacement of vehicle body we observed that the Secondary spring stiffness is the most dominant factor to influence the displacement of Vehicle body followed by Primary Damping Coefficient, Secondary Damping coefficient and Primary spring stiffness. It is observed that influence rate of displacement of vehicle body for the following Secondary spring stiffness is 66.76%, primary damping coefficient is 23.8%, secondary damping coefficient 5% and primary spring stiffness is 4.412%. From the figure 6b, Pareto diagram for Standardized effect on displacement of wheel we observed that the Primary damping coefficient is the most dominant factor to influence the displacement of Wheel followed by Secondary spring stiffness, Secondary Damping coefficient and Primary spring stiffness. It is observed that influence rate of displacement of wheel for the following primary damping coefficient is 40.97%, Secondary spring stiffness is 22.77%, secondary damping coefficient 20.83% and primary spring stiffness is 15.416%.

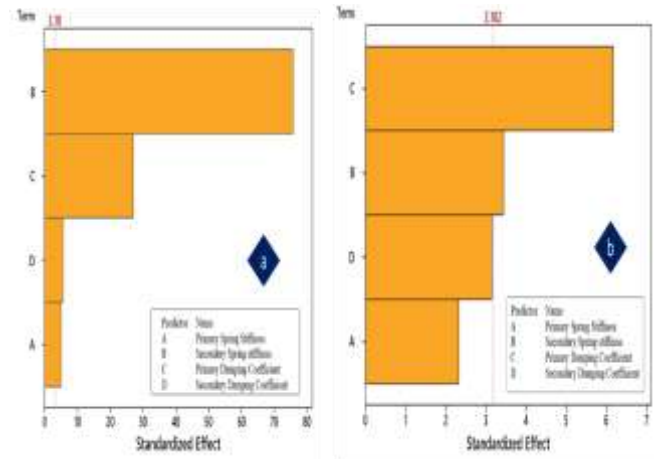


Figure 6. Pareto diagram for a) displacement of Vehicle body and b) displacement of wheel

4. CONCLUSION

Suspension systems can retain the necessary stability and comfort due to the ability of adaptation in real time in compliance with the state of vehicle in real time. In this work the Simulink model has been developed and performed using a six degree of freedom of a railway vehicle system. This model is used to determine and analyse the dynamic response of a railway vehicle with displacement of wheelset in time domain system. Thus maximum values of vertical displacement are obtained. By Sensitivity analysis we obtained various result.

- 66.76 % of vehicle body displacement is primarily influenced by Secondary spring stiffness
- 40.97 % of Wheel displacement is primarily influenced by Primary damping coefficient

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REFERENCE

- 1) Mitra, N.Benerjee, H.A. Khalane, M.A. Sonawane, D.R. Joshi, G.R. Bagul. Simulation and Analysis of Full Car Model for various Road profile on an analytically validated MATLAB/SIMULINK model, IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE), 22-33.
- 2) P.M. Jawahar and K.N. Gupta. Mathematical modelling for lateral dynamic simulation of a railway vehicle with conventional and unconventional wheelset, Mathl Comput. Modelling, 14, 1990, 989 - 994.
- 3) Saurav Talukdar, Anupam Mazumdar, Murukesh Mullasseril, Karuna Kalita and Aditya Ujjwal, Mathematical Modeling in Vehicle Ride Dynamics, SAE International, 2012.
- 4) Kumar, MS, & Jawahar. P.M. Computer Simulation of the Response of a Railway Vehicle

Carbody, Proceedings of the ASME 2002 International Mechanical Engineering Congress and Exposition, 2002, 147-153.

- 5) G. Gu, D. M. Lilley, F. J. Franklin. A structural articulation method for assessing railway bridges subject to dynamic impact loading from track irregularities, *Vehicle System Dynamics*, 48, 2010, 1077-1095.
- 6) M. Hamed, B. Tesfa, F. Gu and A. D. Ball, A study of the suspension system for the diagnosis of dynamic characteristics, 20th International Conference on Automation and Computing, Cranfield, 2014, 152-157.
- 7) Karim H. Ali Abood, Prof. R. A. Khan. Railway Carriage Model to Study the Influence of Vertical Secondary Stiffness on Ride Comfort of Railway Carbody Running on Curved Tracks, *Modern Applied Science*, 5(2), 2010.
- 8) Nangolo N.F., Soukup.J. , Rychikova L., Skocilas J., A combined numerical and modal analysis on vertical vibration response of railway vehicle, *Modelling of Mechanical and Mechatronic Systems, Procedia Engineering*, 96, 2014, 310 – 319.
- 9) Alzbutas, R., & Janilionis, V. The Simulation of Dynamic Systems Using Combined Modelling. *Mathematical Modelling and Analysis*, 5(1), 2000, 7–17.