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Vibration Analysis and the Minimization of Structural Response in High Impact Lateral Loading Environment

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ABSTRACT: Lateral loads act transversely to longitudinal axis of high-rise structures including offshore platforms, and dams etc, the line of action together with that of gravity load produces a resultant line of action, characterized with impending motion ($\sum F = ma \neq 0$) and finite displacement (ie, θ and Δ) which produces vibration of the structural system, an undesirable effect on structural performance and stability. The paper identifies the loading intensity acting on structural system that influence on stability and the importance of accurately predicting structural responses that enables stability within permissible limits and static equilibrium state for performance. Vibration is dynamical structural phenomenon that produce oscillatory motion, expressed by equations, $F = -kx = m\frac{\partial^2 x}{\partial t^2}$, and $-kx/m = \frac{\partial^2 x}{\partial t^2}$. Solution of the equation is a sinusoidal position function x(t) = A cos(wt-φ), where A is the amplitude (or maximum displacement. The study identified that motion of vibratory system can be optimized using principle of minimum potential energy and virtual work method, which express that work done on system undergoing virtual displacement is negligible, ie, $W = F\,\partial x \approx 0$, since $\partial x \approx 0$, an infinitesimal displacement. Damping mechanism are often required to reduce the motion potential, and defined as influence upon a system to reduce oscillation to minimal and insignificant state, which was further enumerated with concept of wave interference ($W_1 + W_2 = 0$, ie, destructive interference). Similarly, damping ratio is the system parameters that varies from undamped ($\xi = 0$), underdamped ($\xi < 1$), critically damped ($\xi =1$) and overdamped ($\xi > 1$). In conclusion, the paper indicated that dynamical tendency is characterized with instability of structural systems with impending motion, hence corresponding vibratory displacement and amplitude of oscillations must be very minimal within structural code permissible limits.

KEYWORDS: Offshore platform, Lateral environmental forces, Vibration, Displacement, Damping, Wave interference.

1.0. INTRODUCTION

Structural loads are forces, moments, or actions, applied to structural system and effect of these loads on structures include stress, strain, deformation, displacement and fatigue. High impact load may cause instability since such condition are not anticipated nor considered during structural design (Thomson, 2003). Impact loads are suddenly applied which has greater effect than gently applied load on structures, likewise vibration produces additional effect because of associated oscillatory motion and displacement amplitudes, which must not exceed beyond maximum magnitude of displacement permitted in design codes and engineering standards. Damped vibration occurs when motion is restraint and the systems energy is gradually dissipated by friction and other resistances (Lazan and Garcia-Raffi, 2022), hence the vibration is reduced gradually to rest in an equilibrium state. Study of vibration is concerned with oscillatory motion of rigid bodies and forces associated with them, the motion is usually characterized with periodic (or cyclic) function, which is a wave-like function that repeats its values at regular intervals. All bodies with mass and elasticity are capable of vibration, hence most engineering machines and structures experience vibration to some degree (Afolabi, 2025), and their design requires consideration of oscillatory behaviors and characteristics (ie, amplitude, frequency, motion etc), to ascertain stability during load application and service period. In relativity principle, a different action must be maximized or minimized as functional to attain stable equilibrium state, therefore to minimized transverse displacement in vibratory system, the action of the applied load on the structure must be optimized in order to evaluate the extrema functions that will make the functional attains a maximum or minimum value, that is, maximum applied force corresponding to minimum displacement and deformation etc.

1.1: Algorithmic sequence of vibration and the oscillatory motion

The motion equation, $F = -kx = m\frac{\partial^2 x}{\partial t^2}$ -- (1)

$$\frac{\partial^2 x}{\partial t^2} = \frac{-k}{m}x = -w^2x$$

$$\frac{\partial^2 x}{\partial t^2} + w^2x = 0 \quad \text{--- (2)}$$

Solution to the differential equation (2) produces sinusoidal position function, ,

. x(t) = A cos(wt + φ)

Structurally, $x(t) \leq x_p$ (x_p = permissible displacement specified in design codes)

A = wave amplitude (ie, maximum displacement)

w = angular velocity = 2π/T

φ = phase constant (or, initial position of oscillation)

T = period of oscillation in SHM is time taken for one complete cycle of oscillation

Thus, x(t) must be minimized during load applications with respect to permissible limits and other structural effects (eg, deformation and fatigue) to ensure static equilibrium state. According to the principle of least action/displacement $x(t) \approx 0$ for expected static equilibrium state of structures, eg, $F = ma = 0$ (or $\sum F = 0$ and $\sum M = 0$).

The expression shows that for adequate performance; structures must remain at rest during load application and therefore restraint from motion

Consequence of higher structural response during impact loading and vibrations include the followings,

High rate of deformation, ie, $\frac{\partial \varepsilon}{\partial t}$

Finite displacement (lateral and torsional)

Low fatigue strength and fracture limit compared with normal structural condition

Rate of strain energy is high, implies vibratory systems consumes high energy for work purposes, which equals the strain energy loss.

ie, $U_r = U_i - W$, and $W = \Delta U$

Structural stability is a characteristic property of engineering systems and a function that describe time dependent of particle-points in geometric space which are in static equilibrium under system of forces (Afolabi and Ibanga (2026). Equilibrium is said to be static, if small externally induced displacement from that state produced an opposing force that returns the body or particle to the equilibrium state, similarly equilibrium is unstable if least displacement produces forces that tend to increase the displacement and possibly motion. Thus, constraint and reactive forces provide resistance to impending motion by keeping the system at rest. Constraints are parameters that provide resistance to continuous or straight movement of rigid body system (Afolabi, 2025), hence condition of constraint motion are dictated by condition of the restraint. Typical examples include periodic motion, circular motion, or static condition of rest during force application. If a system of particles moves parallel to a fixed plane, the system is said to be constraint to planar movement, because of the resistance to motion in the transverse direction. In this case, Newton's laws for a rigid system of N particles, ie, P_i, I = 1, 2,..n, is considered satisfied, because no movement (or, motion) is anticipated or expected in that direction.

2.0. LITERATURE REVIEW

2.1. Dynamics involve the behaviour of structures subjected to dynamics which possess high tendency for motion, and the dynamic analysis is used to determine the behaviour such as, displacement history, time, and modal analysis eg, frequency (Kuznetsov, 2008, and Afolabi, 2020). The difference between the dynamic and static analysis is on the basis of whether the applied action produces sufficient acceleration compared to the structure's natural frequency, and if a load is applied sufficiently slowly, the inertia forces can be ignored and the analysis be simplified as static, otherwise if it varies quickly (relative to the structures ability to respond), the response must be determined with dynamic analysis to evaluate structures mode shapes and frequencies (Bazant, 2000).

$F = ma$, $\quad \frac{F}{m} = a \ \geq 0$

If, a = 0, static equilibrium state and when a > 0, it defines dynamic state

2.2. Increase in the effect of dynamic load is expressed as dynamic amplification factor DAF or dynamic load factor DLF (Chao et al, 2020), defined as

$$DAF = \frac{\partial_{dynamic}}{\partial_{static}} \qquad \text{---- (3)}$$

where δ is the response of the structure due to the applied load

The static equilibrium equation used in the displacement method of analysis is of the form

$$F = k\,x \qquad \text{----- (4)}$$

where the applied force is F, the stiffness is k, and x the displacement.

If the static force is replaced by dynamic or time-varying force F(t), the equation of motion becomes dynamics involving impending motion and displacement, and defined thus,

$$F(t) = m\frac{\partial^2 x}{\partial t^2} + c\frac{\partial x}{\partial t} + k\,x(t) \qquad \text{---- (5)}$$

Where $m\frac{\partial^2 x}{\partial t^2}$ represent the accelerating force, $c\frac{\partial x}{\partial t}$ the motion constraint (or resistance to motion) and k x(t) is the force corresponding to static displacement.

The dynamic equation (Begoni et al, 2012, Chao et al, 2020) must be satisfied at each instant of time during the time interval under consideration, and also the time dependence of the displacements provides two additional forces that resist the applied force in the dynamic equation.

According to Newton's second law of motion that, a particle acted on by force (tongue) moves so that the time rate of change of its linear (angular) momentum is equal to the force (or torque),

$$F(t) = \frac{d}{dt}\left(m\frac{dv}{dt}\right) = m\,\ddot{v}(t) \qquad \text{----- (6)}$$

but the force being restraint from due to the design requirement to remain at rest uses the internal energy to resist the effect of the force to remain in static equilibrium state (ie, $\sum F = ma = 0$)

2.3. Structural performance is characteristic property defined with the strain energy or resilience ($U_r = U/vol$) that provide ability for continuous load application on structures (Silva et al, 2020). A resilient system has ability to recover in a disruptive event, hence it is expected to resist an extreme event with minimal deformation and recovered quickly to the initial state, Mayar et al, 2023 expressed that resilient is characterized by four Rs, defined as,

Robustness – possessing adequate stiffness and stability

Redundancy – having adequate reactive forces to keep the system in static equilibrium

Resourcefulness – design with sustainability and resilience

Recover quickly – governed with limit state parameters and criteria

The total potential energy (π) is the sum of elastic strain energy U, stored in the deformed body and the potential energy (PE) associated to the applied forces

$\pi = U + PE$ ----- (7)

Principle of least displacement (ie, $\Delta s \approx 0$), or more precisely the principle of minimal displacement action, indicates that, displacement of a rigid body must be relatively minimal and negligible, for it to be assumed stationery and/or at rest position (required for structural system and load application). The displacement is at stationary position, when an infinitesimal variation from such position involves no change in energy, (ie, conservation of energy principle)

Thus, $\Delta \pi = \delta U + \delta(PE) = 0$ -- (8)

Virtual work principle states that a body subjected to force application and responses with negligible displacement the work-done is zero.

Ie, $W = F \; \Delta D \approx 0$ --- (10)

since $\Delta D \approx 0$ and negligible that is a virtual displacement

3.0. METHODOLOGY

Offshore structures are subjected to vibration due to their specific location, which possess high dynamical tendency that affects static equilibrium state leading to impending osculatory motion characterized with amplitudes and finite displacement, which is unstable for structural performance, hence the structures are designed with large stiffness and robustness to enable adequate resistance to impending motion and also ensure the structural systems remain in static equilibrium during load application

Ie, $F = ma = 0$ ie, $a = 0$ (for performance of structural systems)

Thus application of restraints forces enables, $a = 0$ in the motion equation (Newtons 3rd law)

Eg, $F - R = 0$ and $F = R$

Where R is reactive force of structural system against impending motion, which reduces over time due to persistence action of F on the structures.

Hence, $F \leq R$ for a stable system

At any given time, a dynamical system has a state representing a point in space, which often is expressed by a vector in geometrical manifold. The condition rule of the system is a function that describes, what future states follow from the current state as a result of continuous action of applied load and the structural response including loss of strain energy. Offshore structures are exposed to aggressive environmental load (eg, wind, wave, current, tides etc) which are dynamical hence the structure must be in dynamic equilibrium with the applied force to remain at rest. For structural performance, the reactive forces are employed to restrained motion and relative displacement of structural systems. The amplitude of oscillation must be minimal and within permissible displacement with respect to structural design codes for comfortable use and performance. Structural stability is fundamental property of dynamical system which means the qualitative behaviour of the trajectories is unaffected by small perturbation. In mathematics, asymptotic is a method of describing limiting behaviours.

Eg, if $a \geq 0$

Then, $a = 0$ describes static equilibrium state

While, $a > 0$ is a motion or dynamical state and unstable structurally

3.1: Vibration of structure is undesirable and evaluated to ascertain that it will not affect the structural performance, and usually conducted by physical inspection and non-destructive testing, the assessment of damage could be determined to identify cracks points and other structural weakness. Similarly, electronic sensors called accelerometers (Lazam and Garcia-Raffi, 2022) can be used to determine vibration force and corresponding oscillatory motion, and analysis of structural defects.

3.2: Damping is an influence upon a vibratory system that can reduce or prevent its oscillation (Lazaro, 2019), by processes that dissipate the kinetic energy of physical systems . Damping ratio is a dimensionless parameter that describes how oscillations in a system decay after a disturbance, since many systems exhibit oscillatory behavior when disturbed from the position of static equilibrium (Thomson, 2003). Damping ratio provides a mathematical means of expressing the level of damping in a system relative to critical damping, it's defined as the system parameter (ξ) which varies from undamped (ie, $\xi = 0$), underdamped ($\xi < 1$), critically damped ($\xi = 1$) to overdamped ($\xi > 1$). Structures are load bearing and due to their functional requirement are classified between critically damped to overdamped ($1 \leq \xi$), and thus can only be tolerate minimal vibration in order not to affect the damping criteria.

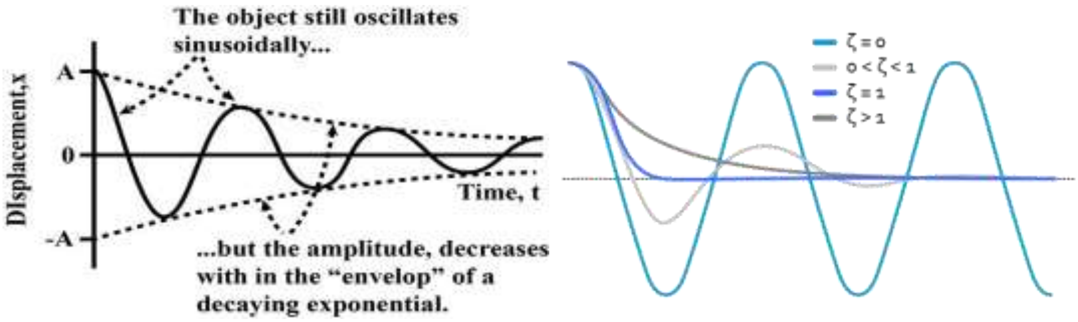

Fig. 3.2: Damping factor of oscillation

For the damped harmonic oscillator with mass m (Arboleda-Monsalvea, et al, 2007), damping coefficient c and spring constant k, the ratio defines the damping coefficient in the system's differential equation to critical damping coefficient.

. $\xi = \frac{e}{e_c}$ = (actual damping)/critical damping -- (19)

A damped sine wave or damped sinusoid is a sinusoidal function whose amplitude approaches zero as time increases, and corresponds to the underdamped case of damped second order systems or underdamped second order differential equations.

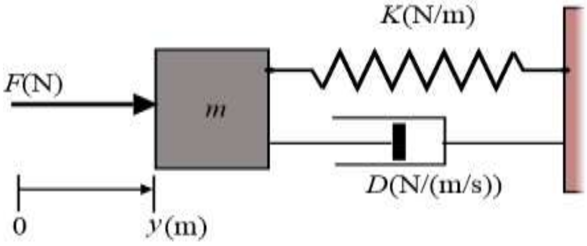

Fig. a

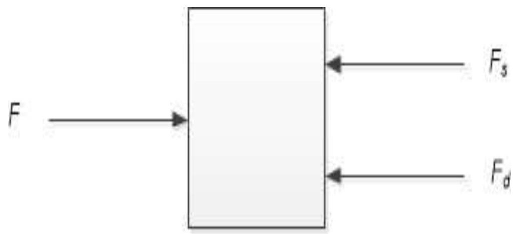

Fig. b (force diagram)

Fig. 3.3: Mass Spring Damper system, F is the applied force, K the spring force and D the damping constant

4.0: ANALYSIS OF STRUCTURAL VIBRATION

4.1: Analysis of structural vibration is required to evaluate structural response to excitations and determine also ability to further fulfill its intended function (Adhikari, 2002). Excitation usually comes from external source and environmental factors like winds, ocean, current and sources internal to the structure including moving loads, reciprocating engines and machinery. Critical structural response to vibrations and oscillatory motions are high rate of deformation, fatigue, and displacements (Δ and θ) which are measurable parameters with permissible limits in structural design.

Applying work-energy theorem, which states that when work done is equal to available strain energy in a structural (ie, W = U)

Considering a cantilever system subjected to longitudinal (gravity loads) and lateral environmental forces, and using the principle

Ie, $F\,\Delta = \frac{1}{2} M\,\theta$

$F = \frac{1}{2}\frac{M\theta}{\Delta}$

Θ the twist and lateral displacement Δ are structural responses in design that depends on the stiffness (EI/l) of a structural member

Considering critical structural response of cantilevered structural system,

if $\theta = \theta_p$ and $\Delta = \Delta_p$ hence $F = F_p$ are the permissible limits

Then $F_e \leq F_p$

Where F_e = environmental (excitation) forces on the system

Similarly using concept of strain energy and least structural action (ie, $\partial \approx 0$)

$U = \frac{1}{2EI}\int Mm\,\partial x$ and

$W = F\,\partial x$ (in least action the displacement is negligible)

Equating equations above, $F\,\partial x = \frac{1}{2EI}\int Mm\,\partial x$

Therefore, increase in EI produces a decrease in value of $F\partial x$, which minimizes the work-done and displacements to optimal acceptable level since F (or environmental force) can be assumed constant.

4.2: Method of waves interference: Waves are propagating dynamics disturbance of one or more quantities in a medium (Espinoza, 2017), also a periodic wave oscillates repeatedly about an equilibrium point at some frequency. Principle of superposition of waves states that when two or more propagating waves of similar type are incident on the same point, the resultant amplitude at that point is equal to the vector sum of the amplitude of individual waves. Thus, two coherent waves are combined by adding the intensities or displacements with considerations of their phase difference and the resultant wave may have greater amplitude (ie constructive interference) or lower amplitude (ie, destructive interference). Energy in an ideal medium is conserved at the point of destructive interference such that when the waves amplitude cancel each other, the energy is redistributed within the medium

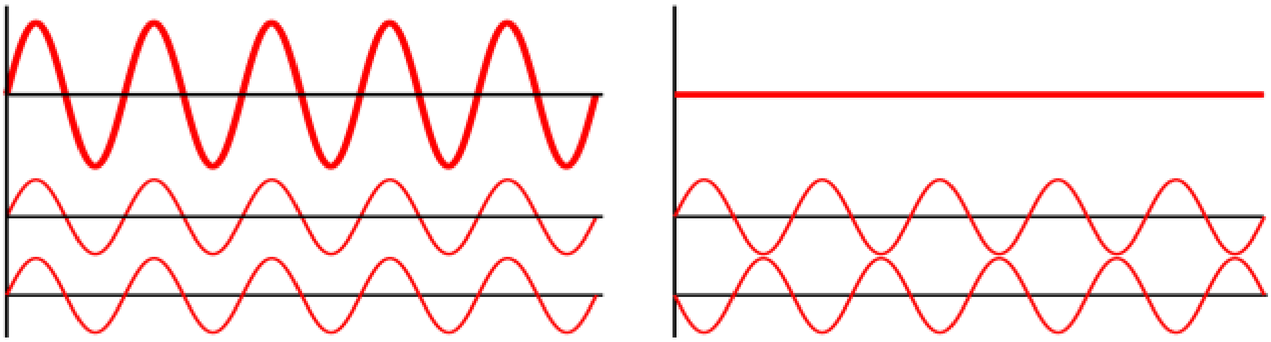

Figure 4.1: Interference of waves

Assuming that equation of sinusoidal wave amplitude traveling to the right along x-axis is expressed as,

$W_1 (x, t) = A \cos (kx - wt)$ ---- (13)

Where A is the peak amplitude, $k = 2\pi/\lambda$ is the wave number and $w = 2\pi f$ is the angular velocity of the wave

Suppose a second wave of the same frequency and amplitude but with different phase is also travelling in similar direction,

$W_2 (x, t) = A \cos (kx - wt + \varphi)$ ---- (14)

Where φ is the phase difference between the waves, the phase of a wave or other periodic function F of some real variable t is an angle-like quantity representing the function of the cycle covered. It is expressed in such a scale that varies by one full turn as the variable t goes through each period

Two waves will superpose and add as follows

$W_1 + W_2 = A (\cos(kx\text{-}wt) + \cos(kx\text{-}wt+\varphi))$ ---- (15)

Evaluating by using trigonometric identity sum of two cosines

$W_1 + W_2 = 2A \cos(\varphi/2) \cos(kx - wt + \varphi/2)$ ---- (16)

The equation represents a wave at the original frequency traveling to the right like its components, whose amplitude is proportional to the cosine of φ/2 (Espinoza, 2017 and Kuznetsev 2008), and further defined as

Constructive interference: if the phase difference is an even multiple of π, and $\varphi = .., -4\pi, -2\pi, 0, 2\pi, 4\pi, ---$. then $\cos \varphi/2 = 1$.

$W_1 + W_2 = 2A \cos(kx - wt)$ ---- (17)

That is, sum of two waves is a wave with twice the amplitude, with corresponding high structural response and displacement, which occurs at resonant frequency of structural system. Resonant is a phenomenon that occurs when an object or system is subjected to extreme force or vibration that matches the natural frequency that produces maximum amplitude and structural response

Destructive interference: if the phase difference is an odd multiple of π and hence $\varphi = .., -3\pi, -\pi, \pi, 3\pi, 5\pi, ..$, and $\cos(\varphi/2) = 0$

$W_1 + W_2 = (0)\, 2A \cos(kx - wt) = 0$ ---- (18)

That is, sum of the two waves is zero, therefore applying waves destructive mechanism can eliminate the influence of wave forces (ie, $F_w < F_p$) on the offshore platforms, by minimizing the lateral forces and corresponding displacements to ensure a stable structural system. Similarly, structural performance are specified usually with respect to tolerable limits which are minimum criteria that enables the structure to remain in static equilibrium, eg, minimal displacement, deformation, fatigue etc beyond which the structure becomes unstable and unsafe for load application (Freitas et al (2016).

5.0: CONCLUSION

Structural performances are defined with condition of static equilibrium and stability that allow for comfortable load application and functionality expected during structural life cycle, and despite the challenging environmental conditions of offshore platforms with high lateral loading acting on structural systems, they must be designed to meet all requirements and criteria for static equilibrium state

Environmental forces (wind, waves, currents, tides etc) acts laterally to the longitudinal orientation of the platform which significantly affects static equilibrium because of resultant effect and line of actions of forces acting on the structure. It is dynamical and produces vibration and motion tendency, with high oscillatory amplitude and finite structural displacement, Response to structural loading, are guided according to structural design codes by specified limits of stress, strain, deformation and displacement to ensure the position of rest (ie, static equilibrium)

Minimizing displacement of vibratory system requires that the action of applied forces on structures be optimized for extrema function and conditions (ie, $\delta s \to 0$), which can be achieved through structural damping such as shoring system to reduce the intensity of the lateral force on the platforms, and also applying appropriate damping mechanism that produces oscillation decay after an excitation, since for least structural action on structural system the damping ratio is within critically damped and overdamped ($\xi \geq 1$).

According to the limit states, high deformation rate is accompanied with low fatigue limit, which provide low structural reliability and therefore to ensure fitness during service period the structure must be designed with adequate robustness and stiffness to resist all forces anticipated on the structural systems and also with routine monitoring and maintenance of the structural systems to ascertain continuity of ensure reliability of load application during structural life cycle.

Vibration testing is an integral test performed on oscillatory system characterized with measurement of maximum vibration level and decay rate, it is used to predict maximum amplitude (ie, displacement), deformation rate and fatigue level. Structural vibration is commonly measured by electronic sensors called accelerometers, which converts acceleration signals to electronic voltage signal, and are evaluated, analyzed and recorded with electronic hardware and compared with permissible magnitude for safety and stability as specified in the design standards. .

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