

Analysis of SSSS and CSCS Rectangular Plates under both Lateral and In-Plane Loads Using Split Deflection Method

Mbahaotu, Godsaveus O^{1.}, Ndububa E. E.^{2.}, Ibearugbulem, O. M^{3.}, Nwosu, Onyebuchi⁴

^{1,2,4}Department of Civil Engineering, University of Abuja

³Department of Civil Engineering, Federal University of Technology, Owerri

Abstract: This paper presents rectangular plate analysis under both lateral and in-plane loads using split deflection method. Thin rectangular plate with primary and secondary in-plane dimensions a and b and thickness t was analyzed in this research. The plates analyzed are ssss plate, simply supported at all edges and cscs plate, simply supported on two opposite edges and clamped on other edges. The total potential energy functional of the plate is minimized with respect to deflection. This gives governing equation, which was solved using split deflection to obtain general equation for deflection. It is also minimized with respect to coefficient of deflection to obtain the formula for the deflection coefficient. The work satisfies the boundary conditions the two plates and obtained unique shape functions for them. Numerical values of the coefficients of deflection and the center deflections of the two plates are determined in the absence and also in the presence of in-plane loads. The results show that the maximum percentage differences recorded for ssss and cscs plates, in the absence of in-plane load, with previous results are 4.86% and 5.15%. It is observed that it is not advisable to work with plate with in-plane load exceeding the critical buckling load.

Keywords: split deflection, rectangular plate, in-plane load, critical buckling load, centre deflection, potential energy

I. INTRODUCTION

The common energy methods used in the past for analysis of rectangular plates for isotropic cases are Ritz, Galerkin and work-error method ([1], [2]). These previous methods used coupled orthogonal deflection function ([3] [4], [5], [6], [7], [8], [9], [2]). It is this coupled form that makes the solution of the resulting governing equation tedious to arrive at, after performing general variation on the total potential energy functional. The few ones that employed split deflection function did not treat a bending of plate problem, which involves both lateral and in-plane loads simultaneously ([2], [9]). This present research is based on the use of split deflection method of rectangular plate analysis, to analyze rectangular plate subjected to both lateral and in-plane load simultaneously. In this method, deflection functions are separated into x and y components of deflection. These components of deflections, which are independent distinct functions, may be represented as “ w ” = $w_x \times w_y$ (where w is deflection, and w_x and w_y are deflection components in x and y direction respectively). Each of the deflection components is product of shape function and coefficient of the deflection. That is $w_x = A_x \cdot h_x$ and $w_y = A_y \cdot h_y$.

II. FORMULATION OF TOTAL POTENTIAL ENERGY EQUATION BASED ON SPLIT DEFLECTION

The plate configuration is as shown on Figure 1.

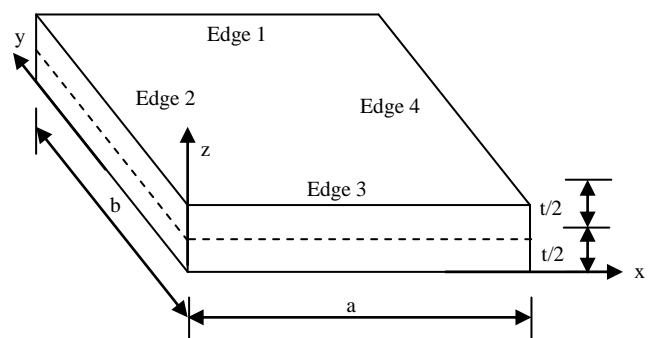


Figure 1: Configuration of thin rectangular plate

The edges of the plate are numbered systematically in anti-clockwise direction. edges 1 and 3 define y direction whereas edges 2 and 4 define x direction. Lengths of the plate along x , y and z directions are a , b and t respectively. This formulation follows the steps: kinematics, constitutive equations, strain energy, external work and total potential energy. Meanwhile, the split-deflection equation is:

$$w = w_x \cdot w_y \quad 1a$$

$$w = A_x \cdot h_x \cdot A_y \cdot h_y = A (h_x \cdot h_y) \quad 1b$$

A. Kinematics

From the third Kirchhoff's assumption, the following hold:

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 \quad \text{That is:}$$

$$u = u_0 - z \frac{dw}{dx} \quad 2a$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0 \quad \text{That is:}$$

$$v = v_0 - z \frac{dw}{dy} \quad 3a$$

Where u_0 and v_0 re integration constants. Substituting equation 1a into equations 2 and 3 gave:

$$u = u_0 - z \frac{d(w_x \cdot w_y)}{dx} \quad 2b$$

$$v = v_0 - z \frac{d(w_x \cdot w_y)}{dy} \quad 3b$$

Differentiation equation 2b and 3b with respect to either x or y resulted the following engineering strain components:

$$\epsilon_x = \frac{\partial u}{\partial x} = -zw_y \left(\frac{d^2 w_x}{dx^2} \right) \quad 4$$

$$\epsilon_y = \frac{\partial v}{\partial y} = -zw_x \left(\frac{d^2 w_y}{dy^2} \right) \quad 5$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z \left(\frac{dw_x}{dx} \right) \left(\frac{dw_y}{dy} \right) \quad 6$$

A. In-plane constitutive equations

Reference [9] gave the in-plane constitutive equations for thin rectangular plates as:

$$\sigma_x = \frac{E}{1 - \mu^2} [\epsilon_x + \mu \epsilon_y] \quad 7$$

$$\sigma_y = \frac{E}{1 - \mu^2} [\mu \epsilon_x + \epsilon_y] \quad 8$$

$$\tau_{xy} = \frac{E(1 - \mu)}{2(1 - \mu^2)} \gamma_{xy} \quad 9$$

Where: E , ϵ_x , ϵ_y , γ_{xy} and μ are Young's elastic modulus, normal strain in x direction, normal strain in y direction, shear strain within xy plane and Poisson's ratio respectively. The normal stress in x direction, normal stress in y direction and the shear stress within xy are respectively designated as σ_x , σ_y and τ_{xy} .

Substituting equations 4, 5 and 6 into equations 7, 8 and 9 where appropriate gives:

$$\sigma_x = \frac{-Ez}{1 - \mu^2} \left[w_y \left(\frac{d^2 w_x}{dx^2} \right) + \mu w_x \left(\frac{d^2 w_y}{dy^2} \right) \right] \quad 10$$

$$\sigma_y = \frac{-Ez}{1 - \mu^2} \left[\mu w_y \left(\frac{d^2 w_x}{dx^2} \right) + w_x \left(\frac{d^2 w_y}{dy^2} \right) \right] \quad 11$$

$$\tau_{xy} = \frac{-Ez(1 - \mu)}{(1 - \mu^2)} \left(\frac{dw_x}{dx} \right) \left(\frac{dw_y}{dy} \right) \quad 12$$

B. Strain energy

The strain energy is commonly defined as given in equation 13 as:

$$U = \frac{1}{2} \int_0^a \int_0^b \int_{-\frac{t}{2}}^{\frac{t}{2}} (\sigma_x \cdot \epsilon_x + \tau_{xy} \cdot \gamma_{xy} + \sigma_y \cdot \epsilon_y) dx \cdot dy \cdot dz \quad 13$$

Substituting equations 4, 5, 6, 10, 11 and 12 into equation 13 gives:

$$U = \frac{Ez^2}{2(1 - \mu^2)} \int_0^a \int_0^b \int_{-\frac{t}{2}}^{\frac{t}{2}} \left\{ w_y^2 \left(\frac{d^2 w_x}{dx^2} \right)^2 + 2\mu \left(\frac{d^2 w_x}{dx^2} \right)^2 \left(\frac{d^2 w_y}{dy^2} \right)^2 + w_x^2 \left(\frac{d^2 w_y}{dy^2} \right)^2 + 2(1 - \mu) \left(\frac{d^2 w_x}{dx^2} \right)^2 \left(\frac{d^2 w_y}{dy^2} \right)^2 \right\} dx \cdot dy \cdot dz$$

That is:

$$U = \frac{D}{2} \int_0^a \int_0^b \left[w_y^2 \left(\frac{d^2 w_x}{dx^2} \right)^2 + 2 \left(\frac{d^2 w_x}{dx^2} \right)^2 \left(\frac{d^2 w_y}{dy^2} \right)^2 + w_x^2 \left(\frac{d^2 w_y}{dy^2} \right)^2 \right] dx \cdot dy \quad 14$$

$$D = \int_{-t/2}^{t/2} z^2 dz \quad 15$$

Substituting equation 1b into equation 14 gives:

$$U = \frac{DA^2}{2} \int_0^a \int_0^b \left[h_y^2 \left(\frac{d^2 h_x}{dx^2} \right)^2 + 2 \left(\frac{dh_x}{dx} \right)^2 \left(\frac{dh_y}{dy} \right)^2 + h_x^2 \left(\frac{d^2 h_y}{dy^2} \right)^2 \right] dx dy \quad 16$$

In terms of non dimensional coordinates, R and Q , equation is rewritten as:

$$U = \frac{DA^2}{2a^4} \int_0^1 \int_0^1 \left[h_y^2 \left(\frac{d^2 h_x}{dR^2} \right)^2 + \frac{2}{\alpha^2} \left(\frac{dh_x}{dR} \right)^2 \left(\frac{dh_y}{dQ} \right)^2 + \frac{h_x^2}{\alpha^4} \left(\frac{d^2 h_y}{dQ^2} \right)^2 \right] ab dR dQ \quad 17$$

Where α is the ratio of length of plate in y direction to length in x direction ($\alpha = b/a$).

In symbolized form, equation 17 is written as:

$$U = \frac{DA^2}{2a^4} \left[k_{xR} \cdot k_{xQ} + \frac{2}{\alpha^2} k_{xyR} \cdot k_{xyQ} + \frac{1}{\alpha^4} k_{yR} \cdot k_{yQ} \right] ab \quad 18a$$

$$U = \frac{DA^2}{2a^4} \left[k_x + \frac{2}{\alpha^2} k_{xy} + \frac{1}{\alpha^4} k_y \right] ab \quad 18b$$

$$k_{xR} = \int_0^1 \left(\frac{d^2 h_x}{dR^2} \right)^2 dR; \quad k_{xQ} = \int_0^1 h_y^2 dQ;$$

$$k_{xyR} = \int_0^1 \left(\frac{dh_x}{dR} \right)^2 dR;$$

$$k_{xyQ} = \int_0^1 \left(\frac{dh_y}{dQ} \right)^2 dQ; \quad k_{yR} = \int_0^1 h_x^2 dR;$$

$$k_{yQ} = \int_0^1 \left(\frac{d^2 h_y}{dQ^2} \right)^2 dQ$$

C. External Work equations

The equations for external works due to lateral load and buckling load were respectively given by Reference [2] as:

$$V_L = q \int_0^a \int_0^b w dx dy \quad 19$$

$$V_N = \frac{N_x}{2} \int_0^a \int_0^b \left(\frac{\partial w}{\partial x} \right)^2 \partial x \partial y \quad 20$$

Substituting equation 1b into equations 19 and 20 and writing the outcomes in non dimensional coordinates and in symbolized form gives:

$$V_L = qA \cdot k_q \cdot ab \implies V_L = qA(k_{qR} \cdot k_{qQ}) \cdot ab \quad 21$$

$$V_N = \frac{N_x A^2}{2a^2} \cdot k_{N_x} \cdot ab \implies V_N = \frac{N_x A^2}{2} (k_{xyR} \cdot k_{xQ}) \cdot ab \quad 22$$

Where:

$$k_{qR} = \int_0^1 h_x dR; \quad k_{qQ} = \int_0^1 h_y dQ;$$

$$k_{xyR} = \int_0^1 \left(\frac{dh_x}{dR} \right)^2 dR; \quad k_{xQ} = \int_0^1 h_y^2 dQ$$

D. Total potential energy equation

Total potential energy equation is the algebraic summation of the strain energy (internal resistant work) and external work. Thus, subtracting equations 21 and 22 from equation 18 gives:

$$\Pi = \frac{DA^2}{2a^4} \left[k_{xR} \cdot k_{xQ} + \frac{2}{\alpha^2} k_{xyR} \cdot k_{xyQ} + \frac{1}{\alpha^4} k_{yR} \cdot k_{yQ} - 2 \frac{qa^4}{AD} (k_{qR} \cdot k_{qQ}) - \frac{N_x \cdot a^2}{D} (k_{xyR} \cdot k_{xQ}) \right] \cdot ab \quad 23$$

III. FORMULA FOR DEFLECTION COEFFICIENT FOR A RECTANGULAR PLATE UNDER BOTH LATERAL AND IN-PLANE LOADS

To get the formula for deflection coefficient for a rectangular plate under both lateral and in-plane loads, the total potential energy equation was minimized with respect to the coefficient. This is otherwise called direct variation of total potential energy. This minimization gives:

$$\frac{d\Pi}{dA} = k_{xR} \cdot k_{xQ} + \frac{2}{\alpha^2} k_{xyR} \cdot k_{xyQ} + \frac{1}{\alpha^4} k_{yR} \cdot k_{yQ} - \frac{qa^4}{AD} (k_{qR} \cdot k_{qQ}) - \frac{N_x \cdot a^2}{D} (k_{xyR} \cdot k_{xQ}) = 0 \quad 24$$

Rearranging equation 24 gave:

$$\frac{AD}{qa^4} = \frac{k_{qR} \cdot k_{qQ}}{k_T - \frac{N_x \cdot a^2}{D} (k_{xyR} \cdot k_{xQ})} \quad 25$$

Where:

$$k_T = k_{xR} \cdot k_{xQ} + \frac{2}{\alpha^2} k_{xyR} \cdot k_{xyQ} + \frac{1}{\alpha^4} k_{yR} \cdot k_{yQ} \quad 26$$

The in-plane load when the plate has buckled under the action of only in-plane load is called the critical buckling load. It is denoted as N_c . For the case of only in-plane load at buckling, the denominator of equation 25 is equal to zero and the in-plane load is replaced with critical buckling load. That is:

$$k_T - \frac{N_c \cdot a^2}{D} (k_{xyR} \cdot k_{xQ}) = 0 \quad 27$$

Rearranging equation 27 gives:

$$\frac{N_c \cdot a^2}{D} = \frac{k_T}{k_{xyR} \cdot k_{xQ}} \quad 28$$

Let the range of the in-plane load be in a range as shown:

$$0 \leq N_x \leq N_c \quad 29$$

Furthermore, let the in-plane load be a fraction of the critical buckling load defined as:

$$N_x = n N_c \quad 30$$

This means that range of n is:

$$0 \leq n \leq 1 \quad 31$$

Substituting equation 30 into equation 25 gives:

$$\frac{AD}{qa^4} = \frac{k_{qR} \cdot k_{qQ}}{k_T - n \frac{N_c \cdot a^2}{D} (k_{xyR} \cdot k_{xQ})} \quad 32$$

Substituting equation 28 into equation 32 gives:

$$\frac{AD}{qa^4} = \frac{k_{qR} \cdot k_{qQ}}{k_T - n \left(\frac{k_T}{k_{xyR} \cdot k_{xQ}} \right) (k_{xyR} \cdot k_{xQ})} \quad \text{That is:}$$

$$\frac{AD}{qa^4} = \beta \cdot \frac{k_{qR} \cdot k_{qQ}}{k_T} \quad 33$$

Where:

$$\beta = \left[\frac{1}{1-n} \right] \quad 34$$

Substituting 1b into equation 32 gives the deflection parameter as:

$$\frac{wD}{qa^4} = \beta \cdot \frac{k_{qR} \cdot k_{qQ}}{k_T} (h_x \cdot h_y) \quad 35$$

IV. NUMERICAL EXAMPLES

Determine the center deflection of ssss and cscs plates under combined actions of lateral load and in-plane load. The components of shape function of deflections to be used are the ones given by Reference [7]:

For ssss plate: $h_x = (\sin \pi R)$; $h_y = (Q - 2Q^3 + Q^4)$

For cscs plate: $h_x = (\sin \pi R)$; $h_y = (Q^2 - 2Q^3 + Q^4)$

With these components of shape functions the stiffness coefficient are calculated and tabulated on Table I

For ssss plate:

$$k_T = 2.396573 \left(1 + \frac{2.000276271}{\alpha^2} + \frac{1.00143}{\alpha^4} \right) \quad 36$$

For cscs plate:

$$k_T = 0.07731 \left(1 + \frac{2.431708407}{\alpha^2} + \frac{5.174055056}{\alpha^4} \right) \quad 37$$

Table I: Stiffness Coefficients For The Two Plate

	SSSS	CSCS
k_{xR}	$\frac{\pi^4}{2}$	$\frac{\pi^4}{2}$
k_{xQ}	$\frac{31}{630}$	$\frac{1}{630}$
k_{xyR}	$\frac{\pi^2}{2}$	$\frac{\pi^2}{2}$
k_{xyQ}	$\frac{17}{35}$	$\frac{2}{105}$
k_{yR}	$\frac{1}{2}$	$\frac{1}{2}$

k_{yQ}	4.8	$\frac{4}{5}$
k_{qR}	$\frac{2}{\pi}$	$\frac{2}{\pi}$
k_{qQ}	$\frac{1}{5}$	$\frac{1}{30}$
$k_{qR} \cdot k_{qQ}$	0.127324	0.021221
$k_{xyR} \cdot k_{xQ}$	0.2428236	0.007833019

At the center of the plates, $R = Q = 0.5$.

For ssss plate:

$$h_x = (\sin 0.5\pi); \quad h_y = (0.5 - 2 \times 0.5^3 + 0.5^4);$$

$$h_x \cdot h_y = \frac{5}{16}$$

$$\frac{wD}{qa^4} = \frac{5\beta}{16} \cdot \frac{k_{qR} \cdot k_{qQ}}{k_T}$$

$$= \frac{5\beta}{16} \times \frac{0.127324}{2.396573 \left(1 + \frac{2.000276271}{\alpha^2} + \frac{1.00143}{\alpha^4}\right)}$$

That is:

$$\frac{wD}{qa^4} = \frac{0.016602353 \beta}{1 + \frac{2.000276271}{\alpha^2} + \frac{1.00143}{\alpha^4}} \quad 38$$

For cscs plate:

$$h_x = (\sin 0.5\pi); \quad h_y = (0.5^2 - 2 \times 0.5^3 + 0.5^4);$$

$$h_x \cdot h_y = \frac{1}{16}$$

$$\frac{wD}{qa^4} = \frac{\beta}{16} \cdot \frac{k_{qR} \cdot k_{qQ}}{k_T}$$

$$= \frac{\beta}{16} \cdot \frac{0.021221}{0.07731 \left(1 + \frac{2.431708407}{\alpha^2} + \frac{5.174055056}{\alpha^4}\right)}$$

That is:

$$\frac{wD}{qa^4} = \frac{0.017156}{1 + \frac{2.431708407}{\alpha^2} + \frac{5.174055056}{\alpha^4}} \quad 39$$

The comparison of the centre deflection for a case where axial load is zero and the result from earlier scholars was by simple percentage difference. The percentage difference equation used is:

$$\%Diff = \text{abs} \left(\frac{W_p - W_E}{W_E} \right) \times 100 \quad 40$$

Where abs means absolute value, w_p means deflection from present study and w_E is the deflection from earlier scholars.

V. RESULTS AND DISCUSSIONS

The result of this present study is presented as follows; Stiffness coefficients for the two rectangular plates are presented on Table I, Centre deflections, w_c (qa^4/D) for ssss and cscs plates, when in-plane load is absent for different aspect ratios are presented on Table II and Table III respectively. In the presence of in-plane, the centre deflections of two rectangular plate were determined the values for various in-plane loads are present on Table IV and Table V for ssss and cscs plates respectively. A critical look at Table II shows a maximum percentage difference of 4.86% when the present

results for ssss plates are compared with those of reference [10]. It is good to note that the method used herein is different from the method used by reference [10]. Again, it is observed that the maximum percentage difference between the results for cscs plate in this present study and those of reference [10] is 5.15%. This is readily seen on Table III. The maximum percentage difference encountered indicates that the difference is not significant, hence justified the credibility of the split deflection method used in this present study. The center deflections of the two plates for various amount of ratio of in-plane load and critical buckling load and for various aspect ratio were determined, the result indicated that in-plane compression load increases deflection of the plates. The increase is gradual from $n = 0$ to $n = 0.3$. Between $n = 0.4$ and $n = 0.5$ the increase is moderate. However, the increase is rapid when n is more than 0.5. Thus, it is advisable to treat a plate as a plate subject to both in-plane load and lateral whenever the in-plane load exceeds 30% of the critical buckling load of the plate. Furthermore, engineers should as much as possible avoid a situation where in-plane load exceeds 50% of critical buckling load. Based on the results of this present study, the split deflection method is justified as a veritable method to analyze rectangular plate under both lateral and in-plane loads.

Table II: Centre Deflection of SSSS Plate

b/a	W_c present study	W_c Reference [10]	Percentage difference
1	0.004148	0.00406	2.17
1.1	0.004974	0.00485	2.56
1.2	0.00578	0.00576	0.35
1.3	0.00655	0.00638	2.66
1.4	0.007277	0.00705	3.21
1.5	0.007955	0.00772	3.03
1.6	0.008583	0.0083	3.40
1.7	0.00916	0.00883	3.74
1.8	0.009692	0.00931	4.09
1.9	0.010178	0.00974	4.50
2	0.010623	0.01013	4.86

Table III: Centre deflection of CSCS plate

b/a	W_c present study	W_c Reference [10]	Percentage difference
1	0.001993	0.00192	3.75
1.1	0.002621	0.00251	4.34
1.2	0.003308	0.00319	3.64
1.3	0.004035	0.00388	3.92
1.4	0.004780	0.0046	3.87
1.5	0.005527	0.00531	4.03
1.6	0.006261	0.00603	3.76
1.7	0.006969	0.00668	4.27
1.8	0.007645	0.00732	4.37
1.9	0.008282	0.0079	4.78
2	0.008880	0.00844	5.15

Table IV; Center deflection of SSSS plate for various aspect ratios and in-plane loads, $w_c (qa^4/D)$

b/a	n = 0	n = 0.1	n = 0.2	n = 0.3	n = 0.4	n = 0.5	n = 0.6	n = 0.7	n = 0.8	n = 0.9
1	0.004148	0.004609	0.005185	0.005926	0.006914	0.008296	0.01037	0.013827	0.020741	0.04148
1.1	0.004974	0.005527	0.006218	0.007106	0.008291	0.009949	0.012436	0.01658	0.024872	0.04974
1.2	0.00578	0.006422	0.007225	0.008257	0.009633	0.01156	0.01445	0.019266	0.028899	0.0578
1.3	0.00655	0.007278	0.008188	0.009358	0.010917	0.0131	0.016376	0.021834	0.032752	0.0655
1.4	0.007277	0.008085	0.009096	0.010395	0.012128	0.014554	0.018192	0.024256	0.036384	0.07276
1.5	0.007955	0.008839	0.009943	0.011364	0.013258	0.015909	0.019887	0.026516	0.039773	0.07954
1.6	0.008583	0.009536	0.010728	0.012261	0.014304	0.017165	0.021456	0.028608	0.042913	0.08582
1.7	0.00916	0.010179	0.01145	0.013087	0.015268	0.018322	0.022902	0.030537	0.045805	0.0916
1.8	0.009692	0.010769	0.012115	0.013846	0.016153	0.019384	0.024230	0.032306	0.048460	0.09691
1.9	0.010178	0.011309	0.012723	0.01454	0.016964	0.020356	0.025446	0.033927	0.050891	0.10178
2	0.010623	0.011803	0.013279	0.015176	0.017705	0.021246	0.026557	0.035410	0.053115	0.10622

Table V. Center deflection of cscsplate for various aspect ratios and in-plane loads, $w_c (qa^4/D)$

b/a	n = 0	n = 0.1	n = 0.2	n = 0.3	n = 0.4	n = 0.5	n = 0.6	n = 0.7	n = 0.8	n = 0.9
1	0.001993	0.002214	0.002491	0.002847	0.003321	0.003986	0.004982	0.006643	0.009964	0.019929
1.1	0.002621	0.002912	0.003276	0.003744	0.004368	0.005242	0.006552	0.008736	0.013104	0.026209
1.2	0.003308	0.003676	0.004135	0.004726	0.005514	0.006617	0.008271	0.011028	0.016542	0.033083
1.3	0.004035	0.004483	0.005044	0.005764	0.006725	0.008070	0.010087	0.013449	0.020174	0.040348
1.4	0.004780	0.005312	0.005976	0.006829	0.007967	0.009561	0.011951	0.015935	0.023902	0.0477805
1.5	0.005527	0.006141	0.006909	0.007896	0.009212	0.011055	0.013818	0.018424	0.027636	0.055273
1.6	0.006261	0.006956	0.007826	0.008944	0.010434	0.012521	0.015651	0.020868	0.031303	0.062605
1.7	0.006969	0.007743	0.008711	0.009956	0.011615	0.013938	0.017422	0.023230	0.034845	0.069689
1.8	0.007645	0.008494	0.009556	0.010921	0.012741	0.015289	0.019111	0.025482	0.038223	0.076446
1.9	0.008282	0.009203	0.010353	0.011832	0.013804	0.016565	0.020706	0.027608	0.041412	0.082825
2	0.008880	0.009867	0.011100	0.012686	0.014800	0.017760	0.022200	0.029600	0.044400	0.088800

REFERENCES

- Njoku, K. O., Ezeh, J. C., Ibearugbulem, O. M., Ettu, L. O., and Anyaogu, L. (2013). Free vibration analysis of thin rectangular isotropic cccc plate using Taylor series formulated shape function in Galerkin’s method. Academic Research International, vol 4, No. 4, pp. 126 – 132.
- Ibearugbulem, O. M., Ibeabuchi, V. I., and Njoku, K. O. (2014). Buckling analysis of ssss stiffened rectangular isotropic plates using work principle approach. Intl. Journal of Innovative Research and Development, vol. 3, issue 11, pp. 169 – 176.
- Ugural, A. C. (1999). Stresses in plates and shells (2nd Ed.). Singapore: McGraw-hill
- Ventsel, E. And Krauthammer, T. (2001). Thin Plates and Shells: Theory, Analysis and Applications. New York: Marcel Dekker
- Szilar, R. (2004). Theories and Applications of plate analysis (Classical, Numerical and Engineering Methods). New Jersey: John Wiley & Sons.
- Ezeh, J. C., Ibearugbulem, O. M., Njoku, K. O. and Ettu, L. O. (2013). Dynamic analysis of isotropic ssss plate using Taylor series shape function in Galerkin’s functional. International Journal of Emerging Technology and Advanced Engineering, vol. 3, issue 5, pp. 372 – 375.
- Ibearugbulem, O. M (2016).. “Note on rectangular plate analysis.” Germany: Lambert Academic Publishing Omni Scriptum GmbH & Co. KG. ISBN: 978-3-330-01636-1
- Ozuluonye, C .G. (2014). “Pure bending analysis of thin rectangular plates using energy method”. Master’s thesis submitted to Postgraduate School,

“Analysis of SSSS and CSCS Rectangular Plates under both Lateral and In-Plane Loads Using Split Deflection Method”

- Federal University of Technology Owerri (unpublished)
9. Ibearugbulem , O. M., Ibearugbulem , C. N., Momoh, H., and Asomugha, A. U.. (2016). “Split – deflection method of classical rectangular plate analysis.” International journal of Scientific and research publication, vol 6, issue. 5, ISSN 2250 – 3153, 2016
 10. Timoshenko, S. P. and Woinowsky- Krieger , S. (1959). “*Theory of plates and shells*”.New York; McGrawHill.