

Structural Appraisal with Optimization for Performance Evaluation Within Limit Criteria

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ABSTRACT: Application of structural load produces stress, strain and deformation over period of load application, that reduces the strain energy (U) of structural system, and also the designed limits are critical factors beyond which performance becomes inadequate and unsafe for continuous load application, which make appraisal an integral aspect of structure's life cycle and to avoid sudden failure or collapse. The study evaluated the significance of structural appraisal using optimization method to determine functionality, fitness and performance. Structures are designed with respect to engineering standards, codes and specifications, also by functionality can only tolerate negligible deformation and minimal displacement (ie $W = F \cdot \delta_s = 0$, if $\delta_s \rightarrow 0$) for the expected limit state and stability. The paper further considered structural failure condition in terms of certain criteria including stress (ie, $\sigma_p \leq \sigma_a$), strain ($\epsilon_p \leq \epsilon_a$) and inelastic deformation, which are factors resulting from work-done to support the applied forces (loads), which consumes available strain energy of structural system leading to gradual reduction over period of time. Evaluation of these parameters using optimization technique provides information on the structural life (ie Strain energy less work done), indicating that strain energy reduces over period of time and as a function $U_r = U_i - \Delta W$, where ΔW is reduction in strain energy over time period. A mathematical optimization is the selection of best solution with regard to some criterion from set of available alternatives (eg, F , σ , ϵ , E and t), that involve maximizing or minimizing a real function for overall benefit of the system. In conclusion, appraisal and engineering reliability are two important parameters with similar outcome and interpretation that defines characteristic performance requirement of engineering system, similarly the reliability function expresses the probability that an engineering system will function under stated conditions for specific time period which provide the assurance of performance.

KEYWORDS: Structure, Appraisal, Deformation, Optimization, Reliability, Performance

1.0. INTRODUCTION

Structural appraisal is involve analyzing the integrity of structures to determine conditions of fitness for load application (Melchers, 1999). Structures are load bearing systems and loss strength over continuous load application due to reduction and energy loss in the system. The strength loss associated with reduced internal energy of structural systems consumed during work-done in sustaining the applied force and to remain in static equilibrium state, ie, negligible displacement. For structural stability displacement must be minimal and insignificant, and observed on structures as deformation leading to fatigue and fracture depending on extent and loading period (Strauss et al, 2019). Similarly materials' characteristic properties identified three distinct stages of components' deformation and response to applied force/load, as follows Elastic deformation: ie, safe for load application Inelastic/Plastic deformation: ie, between safe and unsafe for loading Ultimate load/failed: Not feasible for load application (ie, Failure) Structural failure is damaging condition and loss of structural integrity, which occurs when the material is stressed beyond its strength limits, leading to excessive deformation and fracture. Endurance limit or Fatigue strength describes the property of materials, such as,

the amplitude (or range) of cyclic stress that can be applied to a material without causing fatigue failure (Melchers, 1999). Similarly, "Damage Tolerant Design" is a technique that accommodates all anticipated actions expected on the structure during functional period, and according to limit state philosophy, by allowing an adequate factor of safety, enables a system to continue its intended purpose (Akpan et al, 2015). This allows for an opportunity to inspect component parts periodically for cracks and possibly replace the component, if the observed crack is within tolerable magnitude which does not affect the expected service requirements. Structural health monitoring (SHM) is the engineering procedure of observation, identification and evaluation of damages and flaws, including characterization of strategy for rehabilitation of structures, which enables assurance of continuity of functionality and load application. The process involves observation of structural system over time period, using periodically sampled and dynamic response measurement and equipment (Mori and Nonaka, 2001). Fundamental of structural monitoring is a concept applied widely to various form of infrastructures, such as building, highways, and bridges etc. Collection of damage-sensitive features from the measurements, and statistical analysis are used to determine

the current state and condition of structural systems (Hasni et al, 2017). For long term observations, output of the process are periodically updated, which can provide information regarding the ability of structure to perform functionally and adequately, in the midst of prevailing information and degradation resulting from operational environments. The methods involve the followings, Stress analysis on structural components Degree of deformation and strain assessment

Detecting the existence of damage such as cracks on the structures Locating and identifying the position of flaws (Cracks) Evaluating the severity of the damage; (requires comparison between two states (ie, an initial state, and the present state)).

Evaluation of rehabilitation method and process of structural repairs

1.1: Structures are designed according to requirement of engineering standards and codes, and can only tolerate infinitesimal deformation which often limits the extent of load application bearing the material's characteristic properties Kazaz et al, 2012 & Abdulqader and Atrushi, 2022). Engineering characteristics limits are identified critical threshold of material behaviors within which the material will perform adequately and if otherwise it becomes critical if the material cannot provide resistance to load application and unreliable. It is usually known as permissible, tolerable or design limit, which includes, stress and strain limit, deformation limit, crack limit, fatigue limit etc. Limit State design is a design method that describes “limit state” as condition of a structure beyond which it no longer fulfills relevant design criteria (Kennedy and Aly, 2011). The condition may refer to degree of loading or other actions on the structure, while the criteria refer to structural integrity, fitness for use, durability or other design requirements. A structure designed by LSD is proportioned to sustain all actions likely to occur during its design life and to remain fit for use, with an appropriate level of reliability for each limit state. Limit state design requires the structure to satisfy two principal criteria, which are Ultimate limit state (ULS) and Serviceability limit state (SLS). All design process involves a number of assumptions, and all engineering design criteria have a common goal, which is ensuring a safe structure. (BS 0, 2021) A structure is deemed to satisfy the ultimate limit state criterion, if factored bending, shear and tensile or compressive stresses are below the factored resistances calculated for the section under consideration. The factored stresses referred to are determined by applying magnification factors to the loads on the section, while reduction factors are applied to determine the various resistances of the section. Similarly, a structure may become unfit for use, when it violates the serviceability requirements of deflection, vibrations, cracks due to fatigue, corrosion and fire (Melchers, 1999 and BS 0, 2021)). Therefore, in addition to the ULS check, a service limit state (SLS) computational check must be performed. The aim is to prove that under the action of characteristic design loads (un-factored) and/or

whilst applying certain (un-factored) magnitudes of imposed deformations, settlements or vibration or temperature gradients etc, the structural behavior complies with and does not exceed the SLS design criteria values, specified on the relevant engineering standards. Engineering Standards promote and ensure reliability, efficiency, safety and productivity Kaza et al, 2012). They are formal document for consistent and uniform technical criteria, of engineering methods, process and practices, which are developed from observable characteristic performance of engineering components, equipments and materials over period of time (BS 0, 2021). Design codes, are set of rules and specifications or systematic procedures for design, fabrication, installation and inspection methods, and are prepared, legislated, and adopted by relevant professional institutions to be used as reference document during design and other structural works.

2.0: LITERATURE REVIEW

2.1: Instability is a critical structural condition that occurs when structure lacks ability to provide adequate support to load application (Oriate and Matias, 1996), which is a situation of lower permissible strength of structural component (ie, $\sigma_a > \sigma_p$) or inadequate reactive forces at supports to keep the structure in static equilibrium position ie, $\Sigma F \neq 0$, and $\Sigma M \neq 0$, (Waszczysz N, 1983). The condition cannot be tolerated, otherwise structural failure or collapse will result. Stability is a characteristic property of an engineering system (ie, structures), and a function that describes time dependent of points/particles in geometrical space n static equilibrium ($\Sigma F = 0$, and $\Sigma M = 0$). Equilibrium is said to be static if small externally induced displacements from that state produce force that tend to oppose the displacement and can return the body or particle to the equilibrium state (Zienkiewicz et al, 2005), also equilibrium is unstable if the least displacement produces forces that tend to increase the displacements.

2.2: Lyapunov Stability theory (Pukdeboon, 2011) expressed that If the solutions that start out near equilibrium point x_e stay near x_e forever then x_e is lyapunov stable. Also if x_e is lyapunov stable and all solutions that start out near x_e converge to x_e , then x_e is asymptotically stable. Notion of exponential stability guarantees minimal rate of decay, ie an estimate of how quickly the solutions converge. Lyapunov stability can be extended to infinite-dimensional manifolds, where it is known as structural stability which concerns the behavior of different but “nearby” solutions to differential equations. Input-to-state stability (ISS) applies lyapunov notions to systems with input. ISS is a stability concept used to study stability of nonlinear control systems with external inputs. Structural stability is a fundamental property of a dynamical system which means that the qualitative behavior of the trajectories is unaffected by small perturbations (Jia et al, 2025). Examples of such qualitative properties are numbers of fixed points and periodic orbits. Unlike lyapunov stability that consider the perturbations of initial conditions

for a fixed system, structural stability deals with perturbations of the system itself (Kaza et al, 2012). Stability theory addresses the stability of solutions of differential equations and of trajectories of dynamical systems under small perturbations of initial conditions.

2.3: Failure theories are expressed in form of various criteria, and which sometimes may be specific to certain materials, since engineering materials are defined with specific characteristics property (Gao, 2014). Failure criteria are functions that can be described in terms of stress (ie, $\sigma_a < \sigma_p$), strain (ie, $\varepsilon_a < \varepsilon_p$) or deformation (ie strain space within “plastic yield to ultimate stress limit). Material failure indicates that the material lacks ability to sustain and support significant load increment which is loss of load support capacity of material and structural components. Failure can be examined using variety of scales from microscopic eg, materials constituent particles) to macroscopic (ie, structural components, joints/connections, structures etc, (Akpan et al, 2015). Also failure theory can be used to evaluate the structural integrity (or fitness) of structures using fracture mechanics technique, which involve the study of propagation of cracks in materials. Microscopic Failure, involves the stability of particle points within the continuum of a solid under stress or concentration of stress, and defines failure in terms of crack initiation, propagation and dislocation within the rigid body, including ability to sustain load until fracture occur. The common failure models are the micromechanical failure models which combine the advantages of continuum mechanics and classical fracture mechanics, based on the concept that during plastic deformation, micro-voids nucleate and grow until a local plastic neck or fracture of the inter-void matrix occurs, which causes the coalescence of neighboring voids. Macroscopic Failure is defined in terms of load carrying capacity or energy storage capacity (eg, strain energy) of a structural element (Thi-my-dung, 2020). The classification of macroscopic failure criteria are, Structural deformation Stress concentration, dislocation and cracks Fatigue failure Fracture failure Empirical failure, ie stress-strain analysis and integrity test

3.0: STRUCTURAL OPTIMIZATION

Structural optimization is an optimization technique with aims of finding the best arrangement of structural components, materials strength, and deformation characteristics to achieve certain design objectives under prescribed functionality conditions, including minimizing cost and ensuring durable structure (zheng-zheng et al, 2020) Optimizations refer to acquiring the best outcome under specific conditions, and can be performed in each step of a project life cycle such as design, construction, operation and maintenance. Structures are load-bearing object and implemented through the engineering process of modeling eg, simulation of anticipated forces, actions and material strength etc, (Liang, 2005), design and construction to enable performance and safety during load application. Critical

structural condition during load applications are deformation, cracks, torsion (ie twisting), longitudinal displacement and transverse displacement etc, which are measured with specific limiting service magnitude known as permissible/tolerable conditions or failure mode. Strain energy is the external work done on an elastic member undergoing deformation from the initial unstressed state (Jiang et al, 2022), hence the strain energy reduces over period of time due to work done in resisting the applied force I_e , $U_r = U_i - \Delta W$

and, $U = f(\sigma, \varepsilon, E, t) = f(\text{deformation})$

where ΔW = work-done by the component supporting the applied load over time

U_i = initial strain energy and U_r = strain energy after period of loading

Dynamical Systems are systems in which function describe the time dependence of point in a geometrical space, and at any given time, a dynamical system has a state given by a tuple of real numbers (ie, a vector) that can be represented by a point in an appropriate state space (Bacerra et al, 2017). Evaluation rule of dynamical system is a function that describes what future states follow from the current state, which often is deterministic, that is for a given time interval only one future state follows from the current state. However, some systems are stochastic in that random events also affect the evolution of the state variables. Dynamical systems theory is the mathematical approach that describe the behavior of complex dynamical systems, usually by employing differential equations or difference equations. When differential equations are used the theory is called Continuous dynamical systems and Discrete dynamical systems when difference equations are employed (Farahani et al, 2024). In mathematical dynamics, discrete time and continuous time are two alternative frameworks within which to model variables that evolve over time.

3.1: Mathematical Optimization, is the selection of best solution with regard to some criterion from set of available alternatives (Zheng-zheng et al, (2020). Therefore, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set to compute the value of the function. More generally, optimization involves finding “best available” values of some objective function, given a defined domain (or input), including a variety of different types of objective functions and different types of domains (Liang, 2005).

An optimization problem can be represented in the following way,

Given a function $f: A \rightarrow R$ (ie, Real number)

Determine an element $x_0 \in A$ such that,

$f(x_0) \leq f(x)$ for all $x \in A$ (minimization), or $f(x_0) \geq f(x)$ for all $x \in A$ (maximization)

Optimization problem can be divided into two categories depending on whether the variables are continuous or discrete An optimization problem with discrete variables is known as

discrete optimization, in which an object such as an integer, permutation or graph must be found from a countable set. A discrete variable is a variable whose value is obtained by counting, in orderwords, a discrete variable over a particular range of real value, is one for which any value in the range that the variable is permitted to take on, there is a positive minimum distance to the nearest other value. A problem with continuous variables is known as continuous optimization in which optimal value from a continuous function must be determined. They include constrained problems and multimodal problems. Constrained optimization is the process of optimizing an objective function with respect to some variables in the presence of constraints on those variables. A constraint is a condition of an optimization problem that the situation must satisfy, and there are several types of constraint, such as primary equality constraints, inequality constraints, and integer constraints. Model selection is the task of selecting a statistical model from set of candidate model, given data in the simplest cases, a pre-existing set of data is considered. However, the task can also involve the design of experiments such that the data collected is well-suited to the problem of model selection.

3.2: Principle of Total Potential Energy – Equilibrium is said to be static if small externally induced displacements from that state produce force that tend to oppose the displacement and can return the body or particle to the equilibrium state. Also equilibrium is unstable if the least displacement produces forces that tend to increase the displacements.

External workdone by forces F_i on linear elastic solid that produces set of displacement D_i along the force “line of action” is defined as,

$$W = \frac{1}{2} \sum F_i D_i = \frac{1}{2} (F_1 D_1 + F_2 D_2 + \dots + F_n D_n)$$

Displacement is a vector, whose length is the shortest distance from the initial to final position of a point P. It quantifies both the distance and direction of motion along a straight line, and defined as the difference between the final and initial position vectors (ie, $\Delta s = S_f - S_i$).

Potential energy (Temur et al, 2017) is associated with forces which act on a body, such that total work done by these forces on the body depend only on displacement, which are the initial and final position of the body in space. The principle of least displacement (ie, $\Delta s \approx 0$), or more precisely the principle of minimal displacement action, indicates that, displacement of a rigid body must be relatively minimal and negligible, for it to be assumed stationary and/or at rest position.

Minimum Total Potential Energy express that that a body (eg, structure), shall deform or displace to a position that locally minimizes the total “potential energy”, with the lost in potential energy being converted to kinetic energy for possible motion and displacement, etc (Temur et al, 2017). Similarly, virtual work principle states that a body subjected to force application and responses with negligible displacement the work-done is zero.

$$\text{Ie, } W = F \Delta D \approx 0$$

since $\Delta D \approx 0$ and negligible that is virtual displacement.

Total potential energy (π) is the sum of elastic strain energy U , stored in the deformed body and the potential energy (PE) associated to the applied forces

$$\pi = U + PE$$

This energy is at a stationary position, when an infinitesimal variation from such position involves no change in energy,

Thus, $\Delta \pi = \delta U + \delta(PE) = 0$ (ie, Conservation of energy principle)

The total potential energy and virtual work principle are necessary to minimized displacement and deformation of rigid body system (Preissner and Vinson, 2003), since any displacement beyond permissible limit will subject the structure to unstable equilibrium condition which may not be comfortable for comfortable load application.

4.0: DEFORMATION OF STRUCTURAL SYSTEM

Deformation is the change, in physical dimension, size and shape, of an object as a result of load application (Ghali and El-Badry, 2012), and are defined as elastic when deformation is temporary and can regain its’ initial form, shape and dimension after the removal of stress. It becomes inelastic over period of continuous loading having sustained permanent set, after which it fractures at the ultimate stress limit. Deformation process, is sequential through Load application, Stress, Strain, Fatigue, Fatigue Strength and Fracture or Collapse, also the deformation beyond certain threshold/limit will make the component unfit for continuous loading, and can lead to failure and eventual collapse of the structure/components (Tamrazyan et al, 2022). Fracture will occur at the ultimate stress limit, when the material cannot be subjected to any increment of strain, rather it breaks, because the microstructure can no more accommodate any strain. Fracture could be ductile (ie, progressive) or brittle (ie, sudden), depending on the extent of the material’s strain before failure. Fracture of solid occurs in two steps, which are (i) Crack formation, and (ii) Crack propagation. Deformation rate, provides a limit on the approach of degradation to the fatigue strength, ie, at fracture, which describes reduction of internal energy over period of time.

$$\text{deformation rate} = \frac{\delta \epsilon}{\delta t}$$

4.1: Structural Integrity testing (Norhasri et al, 2021), is an integral part of the modern construction and engineering procedures and practice, and are implemented as quality control and assurance process. Real-life structures or components may require testing by test situation or in the development of models to determine the possibility that an existing infrastructure can continue to meet the standard and functional performances. Structural testing are generally classified, Non Destructive Test (NDT), and Destructive Test (DT). Non-destructive tests are implemented for evaluation of existing facilities, to determine the present capability, using both analytical stress testing and experimental

procedures, while destructive test requires testing to failure stress, and are useful as strength data analysis (Ali, 2023). Failure analysis process relies on collecting failed components for subsequent examination of the cause or causes of failure using a wide array of methods, especially microscopy and spectroscopy. There are several useful methods to prevent product failures occurring, such as failure mode and effects analysis (FMEA) and fault tree analysis (FTA) methods which can be used during prototyping to analyze failures (Zuniawan, 2020). Failure mode and effects (FMEA) is the process of reviewing as many components, assemblies, and subsystems as possible to identify potential failure modes in a system and their causes and effects. For each component, the failure modes and their resulting effects on the rest of the system are recorded in specific FMEA worksheet. FMEA (Abdel and Fayek, 2010) can be a qualitative analysis, but may suggest a quantitative basis when mathematical failure rate models are combined with statistical failure mode ratio database. FMEA is an inductive reasoning (forward logic) single point of failure analysis, and is a core task in reliability engineering, safety and quality engineering. A successful FMEA activity helps identify potential failure modes based on experience with similar products and processes, or based on physics of failure logic (Zuniawan 2020), and widely used in development and manufacturing industries in various phases of the product life cycle.

5.0: CONCLUSION

5.1: The reliability function is theoretically defined as probability of successful performance at time t , denoted as $R(t)$, and estimated from detailed analysis of previous data sets, or through reliability testing and modeling. and maintainability. The goal of reliability assessment is to provide a robust set of qualitative and quantitative evidence that the use of a component or system will not be associated with unacceptable risk. Quantitative risk assessment requires calculations of two components of risk (R), ie, the magnitude of the potential loss (L), and the probability (p) that the loss will occur. An acceptable risk is a risk that usually can be tolerated, thereby implying satisfactory appraisal of the system

5.2: Structural damage is considered as changes to the condition of the material and/or geometric properties of a structural system, including changes in boundary conditions and system connectivity, which could adversely affect the system's performance, such as, load sustainability etc. Deformation data must be checked for statistical significance, and also checked against specified limits, and reviewed to ascertain it is within specific the limit, and if otherwise implies potential risks and possibility of rehabilitation measures

5.3: Cracks are degradation and damaging conditions of concrete structures, which commence from a stress

concentration point within the rigid body and propagate, grow into expanded openings on the surface of the structural components. Crack initiation and propagation enhances fracture, and the nature by which the crack propagates through the material gives great insight into the mode of structural failure. Cracks are initiated through dislocation, that is, a crystallographic defect, or irregularity within a crystal structure of metals or compounds, and can be visualized as being caused by the termination of “plane of atoms” in the middle of a crystal (ie, structural dislocation). Cracks can further be classified, as either active or dormant. If active, they show movement in direction, width, and/or depth over period of time, and when dormant remain unchanged

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