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Mathematical Model for a Two-Tier, Two-Storage-Point Inventory Management System with Lateral Stock Transfer

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ABSTRACT : In our study of the mathematical model for a two-tier, two-storage-point inventory management system with lateral stock transfer, we analyze the parameters required to minimize total inventory management costs and assess the impact of lateral stock transfers between the two storage points. We asked whether stock transfers between storage points can contribute to minimizing total costs and avoiding stockouts.

Lateral stock transfers between storage points could contribute to minimizing total inventory management costs and avoiding stockouts or shortages at the storage points. We first analyzed the inventory management system in the case where there is no lateral stock transfer, and then we studied the case where there is lateral stock transfer, with or without constraints, using the (R,S) inventory management model. The originality of this article lies in the fact that a numerical application was developed using Matlab software, allowing us to analyze the various parameters. It follows that the total management cost is higher in the case of management without lateral transfer than in the case with lateral stock transfer. The contribution of cross-docking between retailers is important to avoid the risk of stockouts and contributes to improving customer satisfaction rates.

KEYWORDS: Inventory management, Levels and storage points, Lateral transfer or stock cross- docking.

1. INTRODUCTION

The inventory distribution network we study comprises two levels and two storage points. [7] The first level concerns the storage sites, while the second level houses the central warehouse. Inventory distribution starts from the upper level (central warehouse) to the lower level (storage points). (See Figure 1). Inventory is a set of products or items, raw materials, supplies, packaging, waste, weapons, and ammunition, kept in a warehouse for later use. [6]

Inventory can also be defined as a reserve of products or goods to be used at critical times. Inventory is also defined as a set of products or goods kept for more or less immediate future use and allows customers to be supplied as and when they need them. Inventory includes a physical flow of finished or semi-finished products, ready for sale. [1]

We assume that the central warehouse has an unlimited amount of stock and can serve retailers (storage sites) without risk of shortages. We conduct our study using the (R,S) management policy, where R is the order release period and S is the maximum stock or replenishment level specific to each site.

Inventory management is primarily concerned with observing and analyzing the company's incoming, internal, and outgoing flows with the aim of seeking a balance between production and consumption in an uncertain environment.[4]

The goal of inventory management is to maximize a company's profitability while minimizing the total cost of inventory management and satisfying customer service requirements.[1]

Service level occurs when demand is not known with certainty. It is defined as the probability that a request will be satisfied. The perfection of customer service level, which can also be defined as the ratio between the number of products supplied and the total inventory quantities requested by customers, was studied.

For a system consisting of two stocking points, the unevenness of stocks between the stocking points is due to random demand. Since one of the two stocking points could have a surplus of stock while the other is facing a shortage, cooperation between these stocking points could be one solution to the problem and avoid any stockouts. The main role of this collaboration is to pool stocks to overcome uncertainties related to customer demand.

We assume that retailer demand is independent and identically distributed (i.i.d.). The methodology used is based on the periodic revision policy (R,S); at the end of each period R, an order is issued to bring the stock back to the replenishment level S. We used the normal (Gaussian) distribution $N(\Delta,\Delta)$, where Δ is the mean of the demand and Δ is the standard deviation of the demand. Using probability density and the normal distribution function, we established the various mathematical expectations involved in the formula for the total cost of inventory management. We partially derived the total cost of inventory management and obtained two equations. Solving these equations using Matlab software allowed us to calculate the inventories that minimize the total cost. Finally, we analyzed the various inventory management parameters.

Our paper is structured as follows: apart from the introduction and conclusion, our study includes two key points:

- Inventory management in a distribution network consisting of a central warehouse and two storage sites.
- Numerical application.

2. LITERATURE REVIEW

We present, in summary, some studies that have addressed multi-echelon and multi-site inventory management.

1. In [BIRI Anthony and All, 2017], the authors analyzed the use of lateral stock transfer between two stocking points supplied by a supplier and examined the impact of this lateral stock transfer between the two stocking points. They showed that lateral stock transfer effectively contributes to maximizing the profit of each stocking point as well as the total profit of the stock distribution network. They insisted that the two stocking points should collaborate fully to achieve the objective: maximizing total profit.

2. The approach presented by [MOALLA Z. BANROUM and TLILI. M., 2010] addressed the issue of inventory management with lateral stock transfer between stocking points in a multi-site, multi-echelon distribution system in which stocking points manage their stocks using (R,s,S) inventory. Stocking points face random customer demands and place orders with distribution centers. When there is a stock shortage at one of the storage points, it carries out an emergency lateral transfer to avoid this stock shortage. The objective is to design an inventory management model that can take into account lateral transfer for this distribution system. The model has three elements: inventory optimization, lateral stock transfer rules, and inventory allocation rules. They solved the problem using Monte Carlo simulation using Crystal Ball software.

3. In [BOUCHERY Yann and Asma Ghaffari; 2012], the authors studied the optimization of a network with two serial inventory levels. The external supplier supplies batches of inventory to the upper level, which in turn supplies inventory to the lower level, which will satisfy consumer demand. The objective was to minimize the total logistics cost of the network, consisting of ordering costs and storage costs at both levels, while examining the impacts related to environmental and social issues.

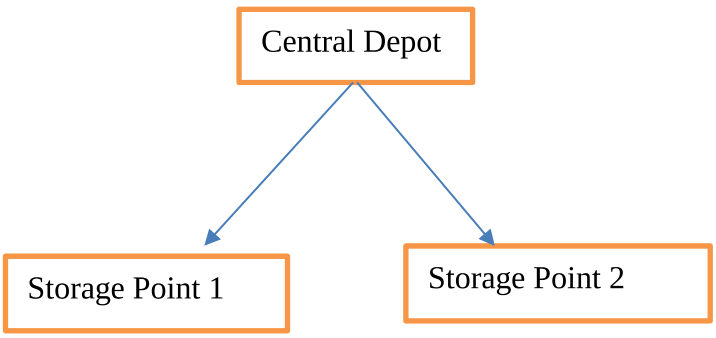
Our contribution aims to optimize the objective function by determining inventories s1*, s2* using Matlab software (see 5. Numerical Application).

3. INVENTORY MANAGEMENT IN A DISTRIBUTION SYSTEM CONSISTING OF A DISTRIBUTION WAREHOUSE AND TWO STORAGE POINTS

The distribution system under study consists of a central warehouse and two storage points (site 1, site 2) that supply goods to customers, having independently placed requests at storage points 1 and 2. We assume that customer demand Di ($i = 1, 2$) follows a normal distribution with a given mean μ_i ($i=1,2$) and standard deviation σ_i ($i= 1, 2$)

Each storage point satisfies the buyers' order Di ($i = 1, 2$). In the event of a shortage or threat of shortage, it can request a quantity Xij of goods from the other storage point. This is the lateral transfer between storage point i, which has a surplus of stock, and storage point j, which faces a shortage or threat of stockout. This transfer is carried out according to the rules concerning the lateral transfer cost, the storage cost, the ordering cost, and the stockout cost.[1] Lateral stock transfer will only be beneficial if the delivery time between storage points 1 and 2 is shorter than the delivery time between the central warehouse and the storage points; otherwise, an order is placed from the central warehouse.

We also note that customers are only in contact with the storage sites and not with the central warehouse. We used the (Ri,Si) policy, the periodic review policy. The procedure for this policy is as follows: order a variable quantity of goods that can bring the stock position up to the replenishment level Si at each period Ri.



4. MATHEMATICAL MODELING OF THE PROBLEM WITHOUT CONSIDERING THE SERVICE LEVEL AT THE STOCKING POINTS

4.1 Problem description

The primary objective of the mathematical modeling of this problem is to determine the optimal inventory level if (i= 1, 2) at both stocking points that minimizes the total inventory management cost.

The total inventory cost is the sum of various costs: ordering costs, transshipment or lateral transfer costs, storage costs, and stockout costs.

$$C_T(s) = \sum_{i=1}^{2} c_i E(Q_i) + \sum_{i=1}^{2}\sum_{j=1}^{2} c_{ij} E\left(X_{ij}\right) + \sum_{i=1}^{2} c_{si} E(I_i^+) + \sum_{i=1}^{2} c_{ri} E(I_i^-) \qquad (1)$$

Where:

- c_i E(Q_i) is the average ordering cost, E(Q_i) is the average quantity ordered by stocking point i, and ci is the ordering cost per unit.
- is the average lateral transfer cost, E(Xij) is the average quantity transferred from stocking point i to stocking point j, which is facing the inventory shortage, and is the lateral transfer cost per unit.
- is the average carrying cost at stocking point i, is the average available inventory held at stocking point i, and is the carrying cost per unit.
- is the average shortage cost at stocking point i, is the average shortage inventory at stocking point i, and is the stockout cost per unit.

4.2 Variables and Parameters Used

1. Parameters

c_i : Ordering cost per unit of goods, per period at site i (i=1,2), expressed in monetary units;

c_{si} : Storage cost per unit of goods, per period at site i (i=1,2), expressed in monetary units;

c_{ri} : Stockout cost per unit of goods at stocking point i (i=1,2), expressed in monetary units/unit of goods/period;

c_{ij} : Transshipment cost from stocking point i to stocking point j, as indicated;

$C_T(s)$: Total management cost per period, expressed in monetary units;

β_i^{av} : Rate or probability of customer satisfaction at stocking point i, before stock transfer;

β_i^{ap} : Rate or probability of customer satisfaction at stocking point i, after stock transfer;

P_i^{av} : Probability of non-stockout or stock shortage cycles during a period R at stocking point i, before transshipment or lateral stock transfer;

: The probability of non-stockout cycles or shortages during period R at storage point i, after cross-docking or lateral stock transfer.

2. Variables

We denote by:

Qi: The quantity of items ordered by each storage point i from the central warehouse;

I_i^+ : The available stock at storage point i, after lateral stock transfer, i = 1.2.

I_i^- : The shortage stock at storage point i, after lateral stock transfer, i = 1.2.

I_i : The actual stock at storage point i, after cross-docking (i = 1.2), is the difference between the available stock and the shortage stock.

X_{ij} : The quantity of lateral stock transfer from storage point i to storage point j that is experiencing a shortage.

y_i : The demand at storage point i or site i, (i = 1.2).

$f(y_i)$: The probability density of the normal distribution for demand yi, (i = 1.2).

$F(y_i)$: the normal distribution function for demand yi, (i=1,2).

4.3 Complete collaboration.

There is collaboration if the storage point with excess inventory agrees to transfer the entire quantity of excess inventory to another storage point experiencing a shortage problem.

Complete collaboration can be expressed as follows:

a) If ∀i , demand is less than or equal to available inventory, i.e., then

$X_{ij} = X_{ji} = 0$ ∀i,j = 1,2 , i≠j, this shows that there is no lateral transfer of inventory.

b) If, i.e., storage point j is in shortage or out of stock due to demand Dj while storage point i has excess inventory, the quantity transferred from site i to site j is:

$X_{ij} = \min(s_i - D_i, D_j - s_j)$ ∀i,j = 1,2 , i≠j

Where: $D_i = y_i$ and $D_j = y_j$

4.4 Expressions of the expected values used in this study.

1. The expected value of transshipment or lateral transfer between storage points i and j.

$$E(X_{ij}) = \int_{y_i=0}^{s_i} F_i(y_i)[1 - F_j(s_i + s_j - y_i)]dy_i \qquad (2)$$

Where: and are the actual inventories at storage points i and j.

2. The expected value of available inventory after transshipment.

$$E(I_i^+) = \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i \qquad (3)$$

3 The mathematical expectation of stock shortage after transshipment.

$$E(I_i^-) = E(y_i) - s_i + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j=0}^{s_j} F_j(y_j)dy_j + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_i + s_j - y_i)dy_i \qquad (4)$$

Where: $E(y_i) = \int_{y_i=0}^{\infty} y_i f_i(y_i)dy_i$

4 The mathematical expectation of net stock.

$$E(I_i) = E(I_i^+) - E(I_i^-) \qquad (5)$$

Replacing (3) and (4) in (5) we have:

$$E(I_i) = s_i - E(y_i) - \int_{y_i=0}^{s_i} F_i(y_i)dy_i + \int_{y_j=0}^{s_j} F_j(y_j)dy_j - \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_i + s_j - y_i)dy_i + \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i \qquad (6)$$

5 The mathematical expectation of the quantity ordered Qi by site i.

The quantity ordered Qi = si – Ii, from this expression we obtain the expression for the average quantity ordered.

$$E(Q_i) = E(y_i) + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j=0}^{s_j} F_j(y_j)dy_j + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_i + s_j - y_i)dy_i - \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i \qquad (7)$$

4.5 Expression of the Total Cost of Inventory Management

By replacing (2), (3), (4), and (7) in (1) and considering two sites i and j, we obtain the expression of the total cost of management for the network under study.

$C_T(s) = (c_i + c_{ri})E(y_i) + (c_j + c_{rj})E(y_j) + (c_{ri} + c_{ij} + c_i - c_j - c_{rj}) \int_{y_j=0}^{s_j} F_j(y_j)dy_j + (c_{rj} + c_{ji} + c_j - c_i - c_{ri}) \int_{y_i=0}^{s_i} F_i(y_i)dy_i + (c_{si} - c_i - c_{ij}) \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i + (c_{sj} - c_j - c_{ji}) \int_{y_j=0}^{s_j} F_j(y_j)F_j(s_j + s_i - y_j)dy_j + (c_{ri} + c_i) \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_j + s_i - y_i)dy_i + (c_{rj} + c_j) \int_{y_j=s_j}^{s_i+s_j} F_j(y_j)F_i(s_j + s_i - y_j)dy_j - c_{ri}s_i - c_{rj}s_j$

4.6 Calculation of inventories (si*, sj*) that minimize total cost.

We will determine si*, sj* minimizing total cost by partially differentiating (8) with respect to si and sj using Leibniz's rule of derivation.

$$\frac{\partial C_T(s)}{\partial s_i} = (c_i - c_j + c_{ri} - c_{rj} + c_{ij})F_i(s_i) + (c_{si} - 2c_j - c_{ij} - c_{ri})F_i(s_i)F_j(s_j) \int_{y_i=0}^{s_i} F_i(y_i)f_j(s_i + s_j - y_i)dy_i + (c_{si} - c_i - c_{ij}) \int_{y_j=0}^{s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j + (c_i + c_{ri}) \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)f_j(s_i + s_j - y_i)dy_i + (c_j + c_{rj}) \int_{y_j=s_j}^{s_i+s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j - c_{ri} = 0$$

$$\frac{\partial C_T(s)}{\partial s_j} = (c_j - c_i + c_{rj} - c_{ri} + c_{ji})F_j(s_j) + (c_{sj} - 2c_j - c_{ji} - c_{rj})F_i(s_i)F_j(s_j) \int_{y_j=0}^{s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j + (c_{sj} - c_j - c_{ji}) \int_{y_i=0}^{s_j} F_i(y_i)f_j(s_i + s_j - y_i)dy_i + (c_j + c_{rj}) \int_{y_j=s_j}^{s_i+s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j + (c_i + c_{ri}) \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)f_j(s_i + s_j - y_i)dy_i - c_{rj} = 0$$

Since the analytical solution of this system is difficult, we will use Matlab's fsolve function to determine s1* and s*2, which minimize the total inventory management cost. (See 5. Numerical Application)

4.7 Transshipment Contribution Between Storage Points

We will examine two possibilities necessary to detect the contribution of lateral transfer:

Transshipment without constraints and transshipment subject to customer satisfaction constraints.

4.7.1 Optimizing Customer Satisfaction Rate Without Constraints

The primary objective of transshipment is to improve customer satisfaction rates to avoid stockouts at one of the sites.

We will examine:

The probability of stockouts not occurring and the fill rate or customer satisfaction.

- Probability of stockouts not occurring at storage point i after lateral transfer.

$$P_i^{ap} = P(y_i \le s_i) + P(s_i < S_i < s_i + s_j - y_j, y_j < s_j)$$

Où : $P(y_i \le s_i) = \int_0^{s_i} f_i(y_i)dy_i = F_i(s_i)$

The probability of no stock shortage at storage point i before transshipment is:

$$P_i^{av} = F_i(s_i)$$

$$P_i^{ap} = F_i(s_i) + \int_{y_j=0}^{s_j} \int_{y_i=s_i}^{s_i+s_j-y_j} f_i(y_i) f_j(y_j) dy_i dy_j$$

The customer satisfaction rate after transshipment between site i and site j is expressed as follows:

$$\beta_i^{ap} = (\beta_i^{av} + E(X_{ji})) / E(D_i)$$

Où: $\beta_i^{av} = \left[\int_{y_i=0}^{s_i} y_i f_i(y_i) dy_i + \int_{y_i=s_i}^{\infty} s_i f_i(y_i) dy_i \right]$

After integration we have; $\beta_i^{av} = s_i - \int_{y_i=0}^{s_i} F_i(y_i) dy_i$

We have the expression for the filling rate before lateral transfer. Replacing (13),(2) in (12) and rearranging the terms, we have:

$$\beta_i^{ap} = 1 - (E(y_i) - s_i + \int_{y_i=0}^{s_i} F_i(y_i) dy_i - \int_{y_j=0}^{s_j} F_i(y_i) dy_i + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i) F_j(S_i + - y_i) dy_i)/E(y_i)$$

Replacing the expression in parentheses with we have

$$\beta_i^{ap} = 1 - \frac{E(I_i^-)}{E(y_i)}$$

This is the expression of the customer satisfaction rate after transshipment.

4.8 Mathematical Modeling of a Constrained System

The mathematical model of constrained inventory management can be written as follows:

$$C_T(s) = \sum_{i=1}^{2} c_i E(Q_i) + \sum_{i=1}^{2} \sum_{\substack{j=1 \\ i \neq j}}^{2} c_{ij} E(X_{ij}) + \sum_{i=1}^{2} c_{si} E(I_i^+) + \sum_{i=1}^{2} c_{ri} E(I_i^-)$$

Under the constraints

$$\beta_i^{ap} \geq \beta_i$$

Where is the service rate generally set at 99%, [10].

Using (14), the constraint can be expressed as follows:

$$g_i(s_i) = E(I_i^-) - (1 - \beta_i) E(y_i) \leq 0$$

$$E(I_i^-) = (1 - \beta_i) E(y_i)$$

Using the substitution method, the objective function becomes:

$$C_T(s) = \sum_{i=1}^{2} c_i E(Q_i) + \sum_{i=1}^{2} \sum_{\substack{j=1 \\ i \neq j}}^{2} c_{ij} E(X_{ij}) + \sum_{i=1}^{2} c_{si} E(I_i^+) + \sum_{i=1}^{2} c_{ri} (1 - \beta_i) E(y_i)$$

By deriving (16) by the Leibnitz derivation [5], we have the system of equations below:

$$\frac{\partial C_T(s)}{\partial s_i} = (c_i + c_{ij}) F_i(s_i) + (c_{si} - 2c_i - c_{ij}) F_i(s_i) F_j(s_j) + (c_{si} - c_i - c_{ij}) \int_{y_i=0}^{s_i} F_i(y_i) f_j(s_i + s_j - y_i) d\, y_i$$
$$+ (c_i) \int_{y_i=s_i}^{s_i+s_j} F_i(y_i) f_j(s_i + s_j - y_i) d\, y_i = 0$$

$$\frac{\partial C_T(s)}{\partial s_j} = (c_j + c_{ji}) F_j(s_j) + (c_{sj} - 2c_j - c_{ji}) F_i(s_i) F_j(s_j) + (c_{sj} - c_j - c_{ji}) \int_{y_j=0}^{s_j} F_j(y_j) f_i(s_i + s_j - y_j) d\, y_j$$
$$+ (c_j) \int_{y_j=s_j}^{s_i+s_j} F_j(y_j) f_i(s_i + s_j - y_j) d\, y_j = 0$$

Since analytical solving is difficult, we will use the Matlab fsolve function, as before, to find the values si* and sj* that minimize the total management cost. (See 5. Numerical Application)

5. NUMERICAL APPLICATION

We will use the Matlab fsolve function to find the inventories si* and sj* that minimize the total management cost. We will also evaluate the expected values, the customer satisfaction rate, and the probability of not running out of stock.
Input parameters.

TABLE 2: MEAN (μ) AND STANDARD DEVIATION (σ) OF DEMAND.

μ_1	μ_2	σ_1	σ_2
200	75	400	106

Table 3: Costs

Unit cost	**Storage point 1**	**Storage point 2**
c_{si}	1.1	1
c_i	22	20
c_{ri}	4	4
c_{ij}	3	3

5.1 Results after analysis

The table CI - below is the result of the execution of the program using the Matlab software which allowed us to calculate the different parameters. From this table, we designed other tables to better analyze the parameters. The graphics are made with the Excel spreadsheet.

Table 4: The results obtained after the execution of Matlab FSOLVE according to the three models.

	Model without transfer		Model with unconstrained transfer		Model with constrained transfer	
i=1,2	Site1	Site2	Site1	Site2	Site1	Site2
s_i^*	310,9	624,0	226	411	271	494,4
C_{si}			644,31	1295,0	643,1	2166,7
$C_T(s)$	3785,9		1939,3		2809,8	
$E(I_i^+)$	47,18	55,48	72,51	119,35	119	198
$E(I_i^-)$	27,15	31?33	0,96	4,377	0,061	0,3965
$E(D_i)$			150,33	300,24	150,2	300,28
$E(X_{ij})$			7,1324	2,9181	8,076	0,1243
$E(Q_i)$			154,44	296,02	151,6	496,6
P_i^{av}			0,8788	0,8225	0,968	0,9497
P_i^{ap}			0,9623	0,9142	0,991	0,9805
β_i^{av}			0,9742	0,9617	0,995	0,9916
β_i^{ap}			0,9936	0,9854	0,985	0,9987

5.1.1 Comparison of the total costs of the three models.

We will compare the total management costs: a system without transfer, with unconstrained lateral transfer, and with constrained lateral transfer.

Table 5: Total cost for each type of management model

	With transfer	without constraint With transfer	under constraint
Coût total	3785,9	2038,1	2809,8

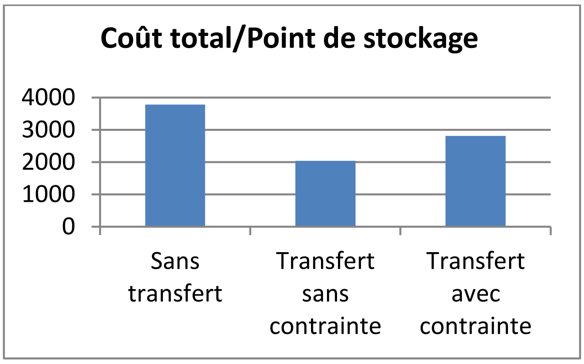


Figure 2: Comparison of the different total management costs for the three cases.

We note that the total cost is higher in the inventory management model without lateral transfer than in the inventory management model where sites transfer their inventory. The total cost in the constrained lateral transfer case is higher than the total cost in the unconstrained transfer case, due to the considerable amount of inventory available in the constrained inventory management model. Lateral transfer contributes to minimizing total cost. Lateral transfer improves total cost by 25.7%.

5.2 Study of satisfaction rates considering the model with unconstrained stock transfer and the constrained management model.

\Table 6: Customer satisfaction rates in the two lateral transfer cases

	Lateral transfer without constraint		Lateral transfer under constraint	
	Site1	Site2	Site1	Site2
β_i^{av}	0,9742	0,9617	0,9947	0,9916
β_i^{ap}	0,9936	0,9854	0,9996	0,9987

β_i^{av}: Customer satisfaction rate before lateral stock transfer.
β_i^{ap}= =βiap: Customer satisfaction rate after lateral stock transfer.

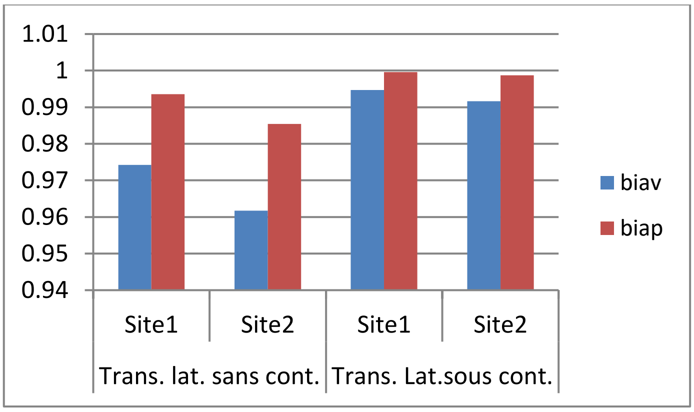


Figure 3: Comparison of customer satisfaction rates before and after lateral transfer

The fill rate or customer satisfaction rate improves after stock transfer in both constrained and unconstrained stock transfer cases. This improvement is less pronounced in the constrained lateral transfer case.
5.3. Study of the probability of stock shortage in the two transfer models with or without constraints.

Table 7: Probability of stock shortage for the two stock transfer models with or without constraints.

	Trans. lat. sans cont.		Trans. lat. sous cont.	
	Site 1	Site 2	Site 1	Site 2
P_i^{av}	0,8788	0,8225	0,9687	0,9460
P_i^{ap}	0,9623	0,9142	0,9913	0,9997

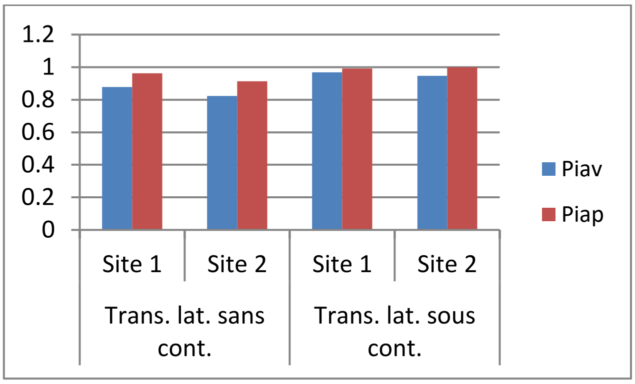


Figure 4: Comparison of the probability of non-stockout in the two cases of lateral transfer, with or without constraints.

Where: Piav is the probability of non-stockout before transfer to site i (Piav) and Piap is the probability of non-stockout after transfer to site i (Piap).

The risk of stockout decreased after the lateral transfer at both sites (1 and 2). Therefore, the lateral transfer reduces the risk of stockout. Lateral transfer in the case of the constrained lateral transfer model significantly improved the probability of non-stockout compared to the case of the unconstrained model.

5.4. Effect of inventory variation on customer satisfaction rate and probability of non-stockout after lateral transfer to site 1. (Unconstrained transfer)

Table 8: Change in customer satisfaction rate and probability of non-stockout

S_i (i=1,2)	**Customer satisfaction rate site 1**	**Probability of non-disruption site 1**
226-411	0,9261	0,7948
246-431	0,9468	0,8278
266-451	0,9625	0,8548
286-471	0,9742	0,8762

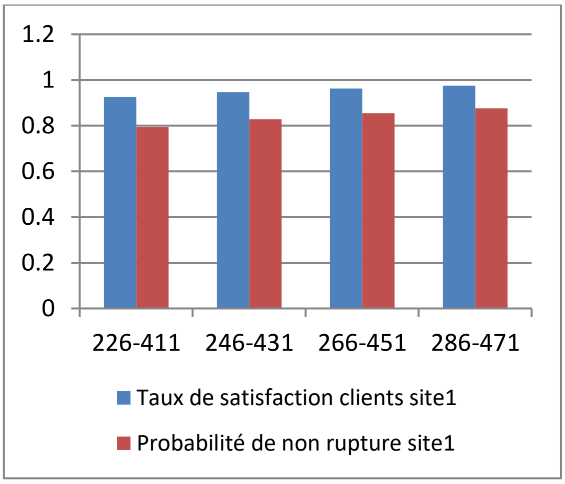


Figure 5: Change in customer satisfaction rate and probability of stockout following the variation in stock at site 1 and site 2.

We note that the increase in available stock at the storage points leads to an improvement in customer satisfaction rate and the probability of stockout at site 1 or storage point 1.

5.5 Change in total cost when available stock varies at both sites

Table 9: Change in total management cost based on the variation in available stock at sites or storage points (1) and (2).

Inventory variation at sites 1 and 2	Total cost
50-150	2085,5
100-200	2050,2
150-250	2036,5
200-300	2028

226-411	2038,1
250-440	2045,6
300-500	2068

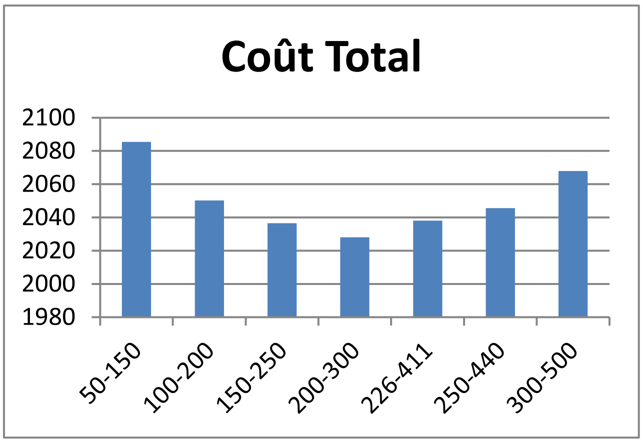


Figure 6: Variation in total inventory management costs as a function of inventory variation available at both sites in unconstrained collaboration.

By varying the available inventory at both sites (1) and (2), from 50-150 to 300-500, we observe that the total cost reaches its minimum value when the inventory is 200 at site 1 and 300 at site 2

5.6 The effect of standard deviation demand variation on total costs

Table 10: Standard deviation demand variation and total inventory cost

Standard Deviation Site1 and 2	Total cost
75-100	1938,4
100-125	1957,4
150-175	2026,8
200-250	2153,4
300-350	2426,6

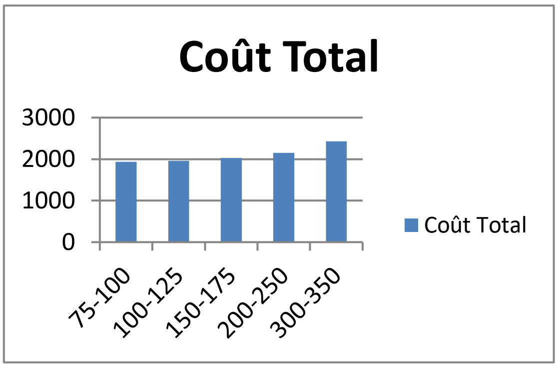


Figure 7: Total management cost variation as a function of demand variation in standard deviation.

We note that an increase in demand in standard deviation leads to an increase in total inventory management costs.

CONCLUSION

We conducted our study on the mathematical model for a two-tier, two-site inventory management system, where retailers place their orders or obtain supplies directly from the central warehouse, which we assumed would be able to meet demand without experiencing shortages or stockouts.

The aim of our work was to examine the parameters necessary to minimize total management costs and to study customer satisfaction rates and the probability of not running out of stock.

The total inventory management cost is lower for collaborating sites than for independent sites.

The risk of stockouts or shortages is lower for sites that perform lateral transfer, while it is significantly higher for independent sites. Lateral stock transfer improves customer satisfaction and the probability of stockouts or shortages.

Demand growth in standard deviations increases total inventory management costs.

Lateral stock transfer significantly improves inventory management in a distribution network composed of two echelons and two storage sites and contributes to minimizing total inventory management costs.

Total inventory management costs reach their minimum value when available stock is 200 at site 1 and 300 at site 2.

We would be more comprehensive if we increased the number of sites and the number of echelons. We have not examined the case where the central warehouse is limited in capacity. In this case, a management policy can be applied to the supplier or the central warehouse.

REFERENCES

1. A. Lazrak, Amélioration des processus de prévision de gestion des stocks dans le cas d'une chaîne logistique des pièces de rechange, *Ecole des mines de Nantes*., 2016.
2. Andrew J. Clark,. Optimal Policies for Muti-Echelon Inventory Problem. RM-2297, Santa Monica, California. *The Rand Corporation*., 2018.
3. Axsäter. Evaluation of unidirectional lateral transshipments and substitutions in inventory systems, European *Journal of Operational Research*.,2003.
4. Y. Bouchery, A. Ghaffari, Z. Jemai, Y. Dallery. Including sustainability criteria into inventory models. *European Journal of Operational Research*, 2012.
5. Biri Anthony and All, Intra _ Firm Inventory Management Model with Transshipments, American Journal and Business Management, 2017.
6. Minner S., E. Silver, and Robb. An improved heuristic for deciding on emergency transhipments. European Journal of Operational Research. 2003.
7. M. Tlili, M. Moalla and J-P. Campagne, Gestion de stocks avec transshipment dans un réseau de distribution multi sites et multi – échelons. *8e Conférence Internationale de MOdélisation et SIMulation - MOSIM’, Hammamet – Tunisie,* 2010.
8. Tagaras,G., Pooling in multi – location periodic inventory distribution systems. Omega International Journal of Management Science, 1999.
9. Wei – Yu and All, Inventory Management with the internet – Based Direct Chanel in a two – Echelon Supply Chain System. The second International Conference on Electronic Business, Taipei Taiwan, 2002.
10. Z. Jemai and All, Analyse d’un problème de tournée de véhicules avec gestion de stock dans un contexte de
11. stock de consignation. 8e Conférence Internationale de Modélisation et Simulation – MOSIM’ 10, *Hammamet – Tunisie,* 2010.