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Details of a Proven Software for the Control of Unmanned Aircraft Systems - Part 1 - Determining the Orientation

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OVERVIEW: The paper describes the software elements that has proven successful in unmanned aircraft vehicles in recent years. The two most important components are the position recognition routines and the flight controller routines. Other elements are of course also necessary for controlling an aircraft, but these are not as mathematically complex. The first of the two parts deals with the basics of attitude recognition. The second part deals with the routines of the controller.

1) OVERVIEW AND NECESSITY OF CONTROL AND ORIENTATION DETECTION

In addition to the question of the energy source, the development of hovering aircraft must always find an answer to the question of how the aircraft is stabilized and controlled. Since the flight characteristics are often unstable, the orientation must be measured and a controller must be used. The author has been building remote-controlled aircraft systems since 1992 and presents the entire flight software here for the first time. An investigation into the achievable flight time was published in 2013 and is available at [3]. An important result is that the flight time does not increase monotonically with the size of the battery, instead of this there is an optimally large battery that leads to a maximum flight time.

Orientation measurement has proven itself in many projects and competitions in recent years, see Figure 1. The software is robust, fast and can also be implemented on simple processors. As an example, the software is described here for two complete copters. Figure 1 also shows the author's first copter. As it was still unregulated, only controlled jumps could be performed with it. All other models were equipped with controllers and sensors and have proven their good functionality in countless hours of flight. It has been observed in past developments that the further development of position detection and controllers is crucial for flight stability.

1992: Tetra — 2008: Big Wood — 2011: Zecke

2013: Wanze — 2014: Songbird — 2023: Scorpion

Figure 1: Examples of systems developed by the author with year of first flight

2) MATHEMATICAL ELEMENTS

a) Quaternions

In [1] it is pointed out that two coordinate systems can be transformed into one another by just one rotation. The orientation vector defines the position of the rotation axis in space. The length of the orientation vector specifies the angle by which the system must be rotated so that all coordinates from the first system can be rotated into the second system.

This point is interesting because it means that rotations do not have to be considered one after the other around several axes. For a rotation or deviation of the position, it is therefore sufficient to determine around which axis and by what amount the rotation must be controlled.

b) Rotary matrix

The rotation around an orientation vector with an angle can be described by rotation matrices.

The complete rotation matrix around the unit vector $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is according to [2]:

$$\begin{pmatrix} \cos\alpha + v_1^2\,(1-\cos\alpha) & v_1 v_2(1-\cos\alpha) - v_3\sin\alpha & v_1 v_3(1-\cos\alpha) - v_2\sin\alpha \\ v_2 v_1(1-\cos\alpha) - v_3\sin\alpha & \cos\alpha + v_2^2(1-\cos\alpha) & v_2 v_3(1-\cos\alpha) - v_1\sin\alpha \\ v_3 v_1(1-\cos\alpha) - v_2\sin\alpha & v_3 v_2(1-\cos\alpha) - v_1\sin\alpha & \cos\alpha + v_3^2(1-\cos\alpha) \end{pmatrix} \quad (1)$$

The unit vector is given with its three components. It must have a length of 1. The rotation angle is also included. This complete rotation matrix contains a large number of arithmetic operations and angle functions, which can lead to long calculation times. It is therefore necessary to investigate whether equation (1) can be simplified for practical applications.

c) Simplification of the rotation matrix for small angles

Before moving on to the simplification for small angles, the expression $(1-\cos\alpha)$ should be considered. This expression is not easy to calculate numerically for small angles. It can be formulated differently to estimate the order of magnitude or for better calculation. A transformation leads to this result:

$$1 - \cos\alpha = 2\sin^2\left(\frac{\alpha}{2}\right) \quad (2)$$

The following simplification for small angles takes into account that the typical rotations of flying objects are smaller than one rotation per second and are therefore less than around 360 °/s. With a measurement and control frequency of around 400 Hz, this results in a rotation angle per time step of less than 1° for each calculation. The following equations are assumed for small angles:

$$\cos\alpha = 1$$
$$\sin\alpha = \alpha$$
$$2\sin^2\left(\frac{\alpha}{2}\right) \ll \sin\alpha \quad \text{therefore} \quad 2\sin^2\left(\frac{\alpha}{2}\right) = 0 \quad (3)$$

Taking into account these three assumptions for small angles, this results in the simplified rotation matrix around the unit vector $\vec{v}$:

$$\begin{pmatrix} 1 & -v_3\,\alpha & +v_2\,\alpha \\ +v_3\,\alpha & 1 & -v_1\,\alpha \\ -v_2\,\alpha & +v_1\,\alpha & 1 \end{pmatrix} \quad (4)$$

This simplification can be used if the rotation per time step of the control is small. The flight controllers described here are only used in these aircraft with these slow rotations, so the simplification can be applied. It does not contain any angle functions and is less computationally intensive overall compared to equation (1) of the complete rotation matrix.

d) The combination of acceleration data and rotation rates

Different sensors for orientation can be used to determine the position. However, as all sensors have measurement errors and systematic errors, single sensors should not be used alone. Acceleration sensors can measure the gravitational vector very well at rest. In a moving system, however, there are also movements and vibrations that the acceleration sensor cannot separate from the gravitational vector. These errors are primarily vibrations. Gyroscopes typically have a long-term drift, which corresponds to a shift of the zero point. If these rotation rates are integrated in degrees per second over the long term, large angles are created. That would simulate a tilted position of the flight system, which does not exist in reality. These errors are primarily drifts and long-term low-frequency shifts.

Two methods are described here. They have proven themselves in the long term. Both are intended to compensate for the errors and shortcomings of the sensors and enable a good description of the orientation. The first method is the complementary filter. It uses a low-pass filter for the acceleration measurements to reduce the vibrations and a high-pass filter for the rotation rate measurements to reduce the low-frequency displacements. The second method is data fusion. Here, a new position is estimated from the known position and the rotation rates, which is referenced by the acceleration data. Both methods are described here.

e) Complementary filter

For an electrical low-pass filter consisting of a resistor and a capacitor, the following differential equation for the current I can be established:

$$I = \frac{U_{in} - U_{out}}{R} = C\,\frac{dU_{out}}{dt} \quad \text{with} \quad \frac{1}{RC} = 2\pi\, f_{limit} \quad (5)$$

Equation (5) can be discretized. After conversion, a formula can be specified that calculates the output voltage as a function of the input voltage:

$$U_{out_{new}} = K\,U_{in_{new}} + (1-K)\,U_{out} \quad \text{with } K = 2\pi\frac{f_{limit}}{f_{sample}} \quad (6)$$

The output voltage changes only slightly during a single time step if K is small. The factor K depends on the cut-off frequency of the low-pass filter and the sample frequency. In the example code, K was set to 0.001. This means that at a working or sampling frequency of around 400 Hz, the cut-off frequency of the low-pass filter is around 0.06 Hz. Higher-frequency vibrations and movements recorded by the acceleration sensors cannot pass through the low-pass filter. These are, for example, imbalances of the propellers or movements of the aircraft due to aerodynamic turbulence.

The following differential equation can be established for an electrical high-pass filter consisting of a capacitor and a resistor:

$$I = \frac{U_{out}}{R} = C\frac{d(U_{in} - U_{out})}{dt} \quad \text{with } \frac{1}{RC} = 2\pi\, f_{limit} \qquad (7)$$

Equation (7) can be discretized. After conversion, a formula can be specified that calculates the output voltage as a function of the input voltage:

$$U_{out_{new}} = \left(U_{in_{new}} - U_{in}\right) + (1 - K)\, U_{out} \quad \text{with } K = 2\pi \frac{f_{limit}}{f_{sample}} \qquad (8)$$

The change in the output voltage depends directly on the change in the input voltage. The factor K is again dependent on the cut-off frequency of the high-pass filter and the sample frequency. In the example code K is still 0.001. This means that at a working or sampling frequency of around 400 Hz, the cut-off frequency of the high-pass filter is around 0.06 Hz. Vibrations and movements that are slower cannot pass through the high-pass filter. This means that a drift or zero point shift of the gyroscope sensors cannot pass through this high-pass filter, as they are significantly slower.

A complementary filter splits a signal or a function f(x) into a part $f(x)_{LP}$, that has passed through a low-pass filter and a part $f(x)_{HP}$, that has passed through a high-pass filter. The sum of the low-pass and high-pass parts is the output function if the cut-off frequency of both filters is the same.

$$f(x) = f(x)_{LP} + f(x)_{HP} \qquad (9)$$

Equation (9) can be shown by adding the amplitude and phase responses of the low-pass and high-pass filters in the frequency domain. It can be seen that the resulting amplitude response is constant 1 and the phase response is constant 0. The result is therefore a neutral filter. Furthermore, it can be shown that the addition of equation (6) with equation (8) leads to equation (9).

f) Data fusion

An alternative method to the complementary filter is data fusion. It combines and weights the information determined firstly from an estimated position and secondly from a measured position. The measured position is based on the direct measurement of the acceleration sensors.

The vector $\vec{h}$ is located in the aircraft and always points vertically upwards from the perspective of the earth. It is exactly opposite to the earth-fixed gravity vector. The acceleration sensors can measure this vector $\vec{h}_{meas}$ directly in the aircraft system.

For the estimated vector $\vec{h}_{est}$ a vector previously determined to be the best vector $\vec{h}_{best}$ is used as the basis and this vector is rotated on the basis of the measurement at the gyroscope sensors, see equation (10).

$$\vec{h}_{est} = \begin{pmatrix} h_{x_{best}} + \alpha_{yaw}\, h_{y_{best}} + \alpha_{roll}\, h_{z_{best}} \\ \alpha_{yaw}\, h_{x_{best}} + h_{y_{best}} + \alpha_{pitch}\, h_{z_{best}} \\ \alpha_{roll}\, h_{x_{best}} + \alpha_{pitch}\, h_{y_{best}} + h_{z_{best}} \end{pmatrix} \qquad (10)$$

The rotation angles α_{pitch}, α_{proll} and α_{yaw} are calculated from the rotation speeds measured by the gyroscope sensors and the time increment between two measurements.

In the final step, the measured vector $\vec{h}_{meas}$ and the estimated vector $\vec{h}_{est}$ are merged into a best, most probable vector $\vec{h}_{best}$. This is based on an averaged mean, see equation (11).

$$\vec{h}_{best} = (1 - K)\, \vec{h}_{est} + K\, \vec{h}_{meas} \qquad (11)$$

The factor K decides which information is weighted more. Flight tests have shown that a value of around 0.001 results in good flight behaviour, which must be based on a good determination of the actual position. Equation (11) is similar to the filters. Similar to equation (6), the vector, $\vec{h}_{meas}$ is given little consideration per time step. This is equivalent to low-pass filtering. High frequencies in the acceleration measurements are less important, as they are not taken into account very much due to long-term summation. The low-frequency drift of the gyroscope sensors is also limited, as the integrated zero-point shifts of the rotations are always referenced by acceleration data and will be removed.

3) ASSEMBLING THE ELEMENTS

Two proven complete programs for controlling hovering aircraft are described here.

a) The multicopter „Wanze"

The first aircraft had a good ranking in a number of competitions, e.g. IMAV 2014, but has not been further developed since 2014. It uses analogue sensors for the rotation rates and accelerations. The complementary filter is also used. The complete source code is available at [4]. At this point, only the parts that are necessary for position determination will be discussed. The components from chapter 1 are used in this chapter. These are equations (4), (6) and (8). The calculation of the position is explained in this thought experiment:

At the time t_0 an observer is sitting in an aircraft. The coordinate x points to the right, y points forwards and z points upwards, see Figure 2.

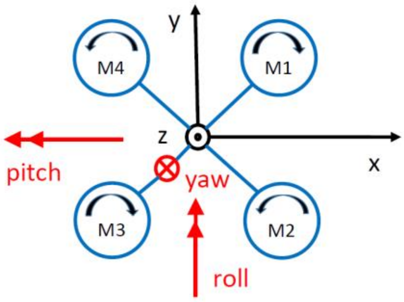

Figure 2: Representation of coordinate system, motor arrangement and directions of rotation

The aircraft is orientated horizontal on the ground. The angular rate sensors do not detect any rotation at the time t_0 and the acceleration sensors do not detect any acceleration in the x and y directions. The acceleration due to gravity is measured in direction z. At this time t_0 an earth-fixed normal vector $\vec{n}$ is defined. This is always perpendicular to the earth's surface in the earth-fixed system.

This corresponds to the three lines below in the setup routine.

```
old_nx= 0; nx_tp= 0; nx_hp= 0;
old_ny= 0; ny_tp= 0; ny_hp= 0;
old_nz= 1; nz_tp= 1; nz_hp= 0;
```

Old values are required for the vector $\vec{n}$ and the low pass filter. These are assumed to be 0 or 1 as the aircraft is orientated horizontal.

The aircraft rotates between the times t_0 und t_1. The rotation is detected by the rotation rate sensors. The sensors are queried in the read_imu routine. The analogue sensors ADXRS610 and ADXL335 were used in the aircraft Wanze.

```
pitch_g_sens= (double(analogRead(0)) - 507) / 1.23;
roll_g_sens= (double(analogRead(1)) - 507) / 1.23;
yaw_g_sens= (515 - double(analogRead(5))) / 1.23;
acc_x= (double(analogRead(4)) - 506) / 155;
acc_y= (515 - double(analogRead(3))) / 155;
acc_z= (double(analogRead(2)) - 506) / 155;
```

This routine depends on the alignment of the sensors and their scaling and calibration. The six lines provide the three rotation rates in degrees / second and the three accelerations as multiples of the earth gravity.

In the first lines of the calc_imu routine, the rotation around the three axes x, y and z is calculated. To do this, the rotation speeds are multiplied by the time and the unit is converted from degrees to radians.

```
imu_dt= double(imu_time - imu_time_old) / 1e6 *
Pi/180;
pitch= pitch_g_sens * imu_dt;
roll= - roll_g_sens * imu_dt;
yaw= yaw_g_sens * imu_dt;
```

The normal vector $\vec{n}$ is rotated by these three components of the rotation. This corresponds to the lines

```
nx= old_nx + roll * old_nz - yaw * old_ny;
ny= old_ny + yaw * old_nx - pitch * old_nz;
nz= old_nz + pitch * old_ny - roll * old_nx;
```

This provides a new normal vector $\vec{n}$. It was only determined on the basis of the old normal vector and the rotation rate measurements. This procedure would be correct if the measurement of the rotation rates were error-free. However, it is subject to long-term drift and must be corrected.

Irrespective of this, the rotation of the aircraft can be recognized by the acceleration values. In the new position, the position of the earth-fixed normal vector can be calculated directly from the three components of the acceleration. This procedure would also be correct if the acceleration measurement was error-free and there were no changes in the translational movement or higher-frequency vibrations.

In the last two paragraphs, 2 independent methods were shown that are suitable for determining the earth-fixed normal vector from the perspective of the aircraft. However, both methods are based on measurements that involve different sensors and measurement errors. The angular rate sensors have a drift, which means that they also display a measured value when there is no rotation. This leads to a false continuous rotation of the earth-fixed normal vector from the perspective of the observer in the aircraft, even when the aircraft is at rest. The acceleration sensors not only detect an inclined position, but also react to translational accelerations and possible vibrations.

The idea at this point is to pass the result of the calculation of the earth-fixed normal vector, which is based on the acceleration measurement, through a low-pass filter. This filter reduces the high-frequency acceleration components that can come from the vibrations and the translational accelerations. The result of the calculation of the earth-fixed normal vector, which is based on the rotation rate measurement, is passed through a high-pass filter. This high-pass filter suppresses the low-frequency components that come from the drift of the angular rate sensors. An exact earth-fixed normal vector can be determined from the low-pass and high-pass components. The errors due to the drift of the angular rate sensors and the influences on the acceleration sensors are very small. The six lines of the low and high pass filter are shown here (tp = low pass, hp= high pass):

```
nx_tp= (1-rx) * nx_tp + rx * acc_x;
ny_tp= (1-rx) * ny_tp + rx * acc_y;
nz_tp= (1-rx) * nz_tp + rx * acc_z;
nx_hp= (1-rx) * nx_hp + nx - old_nx;
ny_hp= (1-rx) * ny_hp + ny - old_ny;
nz_hp= (1-rx) * nz_hp + nz - old_nz;
```

In the calc-imu routine, you can see that the normal vector $\vec{n}$ is normalized to a length of 1. This is necessary if the vector is rotated frequently. Due to numerical inaccuracies, it is possible that its length does not retain the exact length of 1 during a rotation. This unit length must therefore always be restored. As a result of the calc-imu routine, a current earth-fixed normal vector $\vec{n}$ is available, which is an input value for the aircraft's controller.

b) The multicopter „Queen“

The second aircraft has been continuously developed since 2015 and has also taken top places in a number of competitions, e.g. Droneclash 2018, 2019, IMAV 2023. It uses digital sensors for rotation rates and accelerations. Data fusion is also used. The complete source code is available at [5]. At this point, only the parts that are required to determine the position will be discussed. The components from chapter 1 are used in this chapter. These are mainly equations (10) and (11). The calculation of the position is explained in this thought experiment:

At the beginning of t_0 an earth-fixed normal vector $\vec{h}$ is defined. This is always perpendicular to the earth's surface in the earth-fixed system. It is initialized in the line below. It is assumed that the aircraft is horizontal orientated just before the take off.

```
hx= 0; hy= 0; hz= 1;
```

Then the sensors are read, the lines read:

```
readFrom(MPU6050, ACCEL_XOUT_H, toRead, reading);
hx_meas= float)((reading[2] << 8) | reading[3])/2048 - 0.002;
hy_meas= -(float)((reading[0] << 8) | reading[1])/2048 + 0.019;
hz_meas= float)((reading[4] << 8) | reading[5])/2048 + 0.002;
readFrom(MPU6050, GYRO_XOUT_H, toRead, reading);
pitch_g_sens= -(float)((reading[2] << 8) | reading[3])/16.384 + 1.54;
roll_g_sens= -(float)((reading[0] << 8) | reading[1])/16.384 - 0.71;
yaw_g_sens= -(float)((reading[4] << 8) | reading[5])/16.384 + 0.66;
```

This routine also depends on the alignment of the sensors and their scaling and calibration. The MPU6050 was used as the sensor. The eight lines provide the three rotation rates in degrees / second and the three accelerations as multiples of the acceleration due to gravity. It can be seen that values were added at the end of the lines, they were determined in a calibration and compensate for the zero offset of the sensors. For your own developments, the sensor directions must be assigned to the axes of the aircraft. In addition, the sensors must be calibrated before the flight. This is done experimentally using the software in the reference [6].

The next step in the control algorithm is to calculate the rotation during a time step.

```
imu_dt= float(imu_time - imu_time_old) / 1e6;
pitch= pitch_g_sens * imu_dt / Div180Pi;
roll= roll_g_sens * imu_dt / Div180Pi;
yaw= yaw_g_sens * imu_dt / Div180Pi;
```

The normal vector $\vec{h}$ is rotated by the angles pitch, roll and yaw.

```
hx_est= + hx - yaw * hy - roll * hz;
hy_est= + yaw * hx + hy - pitch * hz;
hz_est= + roll * hx + pitch * hy + hz;
```

This is the estimated vector $\vec{h}_{est}$ based only on the measurement with the gyroscope sensors. Finally, this estimate is referenced on the measurements from the acceleration sensors. This is done by data fusion, which takes into account the estimated and measured normal vector in a fixed relation.

```
hx= (1-rx_h) * hx_est + rx_h * hx_meas;
hy= (1-rx_h) * hy_est + rx_h * hy_meas;
hz= (1-rx_h) * hz_est + rx_h * hz_meas;
```

This provides the best result for the normal vector $\vec{h}$. It is used together with the rotation rates in the control algorithm and is also available for estimation at the next time step. The procedure shown here limits the influence of the drift of the gyroscope sensors and filters the high frequencies from the acceleration sensor data.

Two complete programs for controlling unmanned aircraft systems are given in the sources [4] and [5]. It is not to be expected that the untested adoption of software from [4] or [5] in a separate project will lead to success, as the orientation and calibration of the sensors must be taken into account for each separate project. Furthermore, the assignment and direction of rotation of the motors to the axes must be taken into account, see Figure 2. Finally, the program must interpret the existing signals from the remote control correctly. There are a variety of options and protocols for this. The author limits himself to sum signal systems and serial signals from Spektrum or Jeti systems. The control parameters must also be adapted to the aircraft. The control is part of another publication.

A practical comparison of the two methods of attitude determination has shown that calculating the attitude using the data fusion method, e.g. in the Queen project, leads to excellent flight characteristics. The somewhat more extensive calculation using the complementary filter method could not show that it leads to better flight characteristics. Therefore, the data fusion method is used in current and future developments. It is available here as a description and as complete source code. The author is happy to adapt and adjust the software for specific projects for customers.

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