

## Reliability Analysis for Optimization of Galma Dam Reservoir Operations Using an Explicit Stochastic Model

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**ABSTRACT:** Effective handling of water resources in dams is important for guaranteeing consistent water supply, flood management, and food safety. This research introduces a method for assessing the dependability of Galma's dam reservoir operations by utilizing a specific stochastic modeling technique. The stochastic optimization model includes the natural uncertainties in hydrological inflows, water demands, and operating conditions. The stochastic optimization model is created by selecting the probability distribution of two parameters from the Weibull distribution, with a scale factor of 25.3522 and a shape factor of 0.55190. This model is converted into a chance-constrained programming problem to determine the yield for a specified demand. The input discharge levels were optimized to achieve the Maximum-Minimum yields at reliability levels of 25%, 50%, 75% and 85%. The highest amount of water produced was 669.890 Mega Cubic Meters (MCM) with an 85% assurance level, while the lowest amount was 334.2666 MCM at a 25% confidence level. This demonstrates the need to find the best operational strategies that consider trade-offs between different goals and guarantee a specific level of system dependability. The results of this research offer important information for water resource managers and decision-makers to create resilient and sustainable reservoir management plans, especially amidst growing uncertainties from climate change and other environmental influences.

**KEYWORDS:** Optimization, Reliability, Reservoir operation, Inflow, Probability distribution

### INTRODUCTION

Complex societies and economies have evolved and grown in many regions, with water resource infrastructure having been significantly developed. Lund [1], reported that more complex water management problems have arisen that go beyond the optimal operation of a single component or class of water system components. These contemporary water management problems call for a more integrated and comprehensive optimization of water management within a much larger and more diverse economy, and within an institutional structure which is far more de-centralized than traditionally assumed. In this light, effective management of reservoir system has become critical. Reservoir operations are increasingly important in the water cycle [2], [3]. Reservoir operations also support regional growth and development by increasing water availability for various economic sectors, contributing renewable electricity production, and reducing flood risks [4]. The management of river systems by constructing dams has led to need to plan for future activities [5], and to ensure water and energy provision in rapidly advancing African and Asian nations [6]. Rapid changes in climate and society suggest an urgent need to re-operate existing infrastructures [7]. Changes in societal perceptions of natural resources and increasing environmental awareness are modifying and enlarging the number of objectives considered [8]. In addition, changing climate extremes and societal demands amplify and reshape uncertain stressors,

ultimately altering decision makers' preferences and risk perception [9],[10], [11], [12]. Water systems are susceptible to different types of insecurity, such as problems associated with excess and scarcity of water. Thus, using practical operating rules for managing the available volume appears as an alternative to mitigate such difficulties [13].

In the past, disputes involving water (too little, too much, too polluted, and too expensive water), provides opportunities for research on effective management of water resources systems are driven by zeal to improve benefits [14]. The most traditional way of operating reservoirs is the Standard Operating Policy (SOP). The SOP is a relatively simple operation model whose main objective is to meet the demands whenever possible, releasing the maximum amount of water possible and storing only the surplus [15]. The many and different conflicting users upstream and downstream of this system call for studies to promote its optimal operation. However, most of the studies carried out so far only evaluate the performance of the system under different inflow scenarios such as dry, normal and wet scenarios. These scenarios are used as input to simulation or deterministic optimization models and conclusions are taken from the results obtained. Although such procedure is important to evaluate the system under various conditions, but in the views of [16], the operator in practice will never be able to perfectly forecast the actual upcoming scenario. Hence, it is difficult to choose which operating policy to use (dry, normal or wet-

scenario policy). Instead of evaluating the system's performance under pre-defined inflow scenarios it may be advantageous to find operating policies taking the uncertainties into account. Against this backdrop, Using Implicit Stochastic Optimization (ISO) for deriving reservoir hedging rules is an option to mitigate the damages resulting from droughts. The procedure consists of optimizing the operation of the reservoir under different synthetic scenarios of inflows and defining operational rules through regression models, usually relating optimized allocations to water availability [17] [16].

Basically, in the face of climatic flux being experienced globally, the focus of water resource system analysis has turned into defining adequate operation strategies. Better management is necessary to cope with the challenge of supplying increasing demands and conflicts on water allocation while facing climate change impacts, [18]. Implicit Stochastic Optimization (ISO) procedures are techniques that implicitly consider the variability of the irregular space-time distribution of precipitation, and high evaporation rates. [19]. Stochastic optimization techniques account for the inherent variability and uncertainty associated with inflows, demands, and other system parameters [20], [21]

Stochastic programming models provide a framework for incorporating uncertainty into reservoir optimization. These models consider multiple scenarios of inflows, demands, and other uncertain parameters and aim to optimize reservoir operations to achieve robust and efficient solutions. Stochastic programming approaches include two-stage and multi-stage models. However, challenges remain, including the selection of appropriate scenario generation techniques, the accurate estimation of uncertain parameters, and the computational complexity of stochastic optimization models. There is need for new research to focus on advancing the methodologies, integrating real-time data and forecasting techniques, and addressing the practical implementation challenges of stochastic reservoir optimization.

Stochastic optimization is a powerful approach that considers uncertainty and variability in reservoir systems, enabling robust and efficient decision-making. Stochastic optimization models for reservoir systems aim to optimize system operations by considering uncertain parameters such as inflows, demands, and reservoir storage levels. [6], examined the effectiveness of seven stochastic models for determining optimal reservoir operating policies. These models were categorized into implicit stochastic optimization (ISO), explicit stochastic optimization (ESO), and parameterization-simulation-optimization (PSO) approaches. The ISO models include multiple regression, two-dimensional surface modeling, and a neuro-fuzzy strategy. The widely used stochastic dynamic programming (SDP) technique represents the ESO model. The PSO models consist of a variant of the standard operating policy (SOP), reservoir zoning, and a two-dimensional hedging rule. To evaluate the models, a case study was conducted on a single reservoir in northeastern

Brazil, which dams an intermittent river. The comparison also includes the standard operating policy, and deterministic optimization results based on perfect forecasts are used as a benchmark. Overall, the ISO and PSO models outperformed the SDP and SOP models. Moreover, the ISO-based surface modeling procedure and the PSO-based two-dimensional hedging rule demonstrated superior performance compared to the neuro-fuzzy approach. These models typically involve formulating an objective function to maximize benefits or minimize costs, subject to constraints on reservoir storage, release policies, and other operational limitations. Stochastic optimization models can be expressed as mixed-integer linear programs (MILPs) or non-linear programs (NLPs), depending on the complexity of the system and the decision variables involved [12]. Additionally, advanced algorithms such as genetic algorithms, particle swarm optimization, and simulated annealing have been applied to address the computational challenges associated with large-scale stochastic optimization problems [8], [17],[25]. These techniques focused on finding optimal or near-optimal solutions considering the uncertainties in reservoir systems. Stochastic optimization models enable the incorporation of risk analysis to assess the performance and robustness of reservoir operations. Risk analysis techniques, such as value-at-risk (VaR), conditional value-at-risk (CVaR), and chance-constrained programming, are commonly employed to quantify and manage the risk associated with reservoir operations. VaR and CVaR provide measures of the worst-case and expected losses, respectively, under different confidence levels. Chance-constrained programming ensures that the probability of violating system constraints remains below a specified threshold [27]. These risk analysis techniques assist decision-makers in understanding and managing the trade-offs between system performance and risk levels. However, challenges remain in terms of selecting appropriate solution techniques, scenario generation methods, and computational complexity. Future research should focus on refining and advancing these models to enhance their applicability and practicality in real-world reservoir management.

## 2.0 MATERIALS AND METHODS

The study area is located in Kubau local government area of Kaduna state, Nigeria. It geographical coordinates lies between latitudes 10° 48' 45" to 10° 48' 49" and longitudes 8° 23' 9" to 8° 22' 11", (Figure 1). Galma dam is drained by its numerous tributaries. These tributaries among others include river Baki, Anchau and Danwata. The Galma river catchment area belongs to the north eastern part of Kaduna River basin which borders the Chad basin to the north. Rainfall Data used for the study was obtained from Nigerian Meteorological Agency (NiMET) as show in Figure 1, its depict the rainfall pattern for 22 years (1971-2015) The rainfall contributes to the reservoir of the dam through run off, and it serve as the principal input data for the period of the study under review

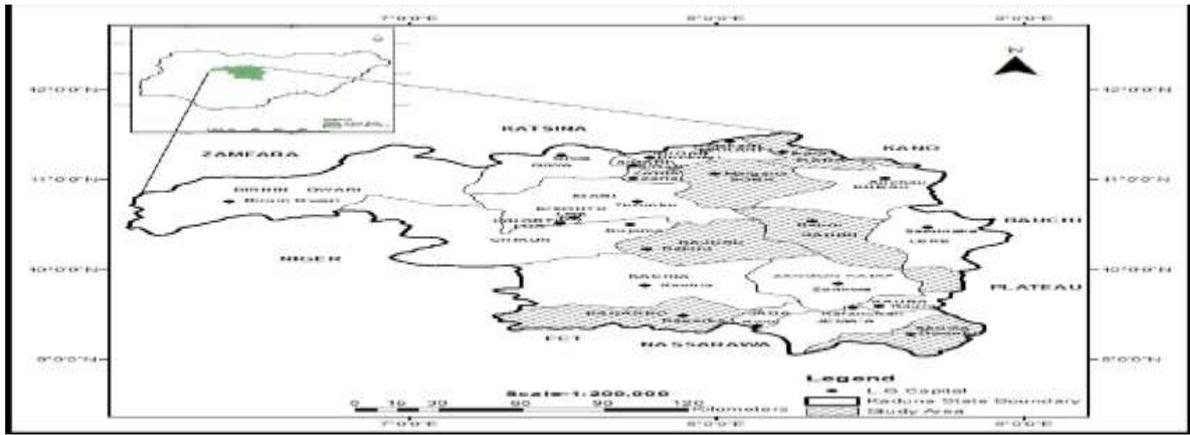


Fig 1: Map of Kaduna State Showing study Area

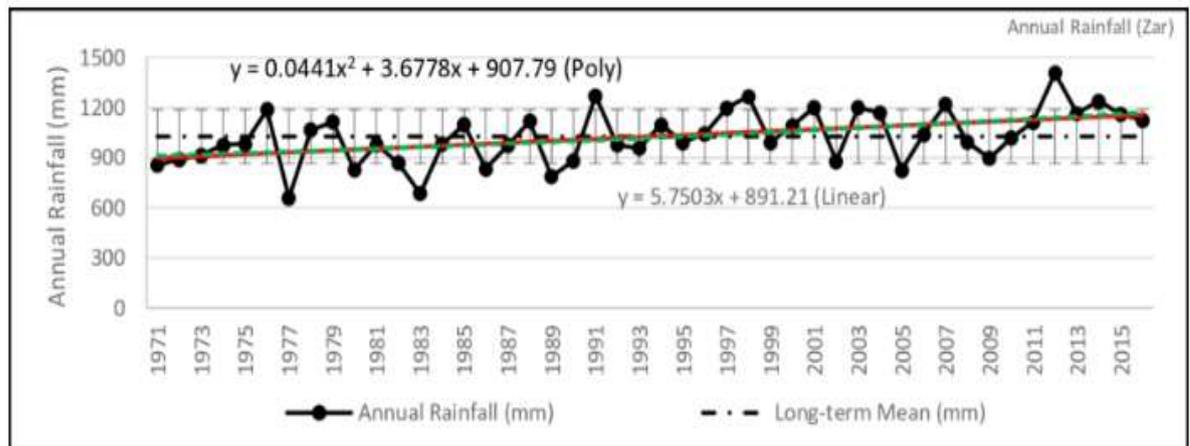


Fig 2: Annual rainfall data for Kubau

The flow chart algorithm for the study is presented in Figure 3.

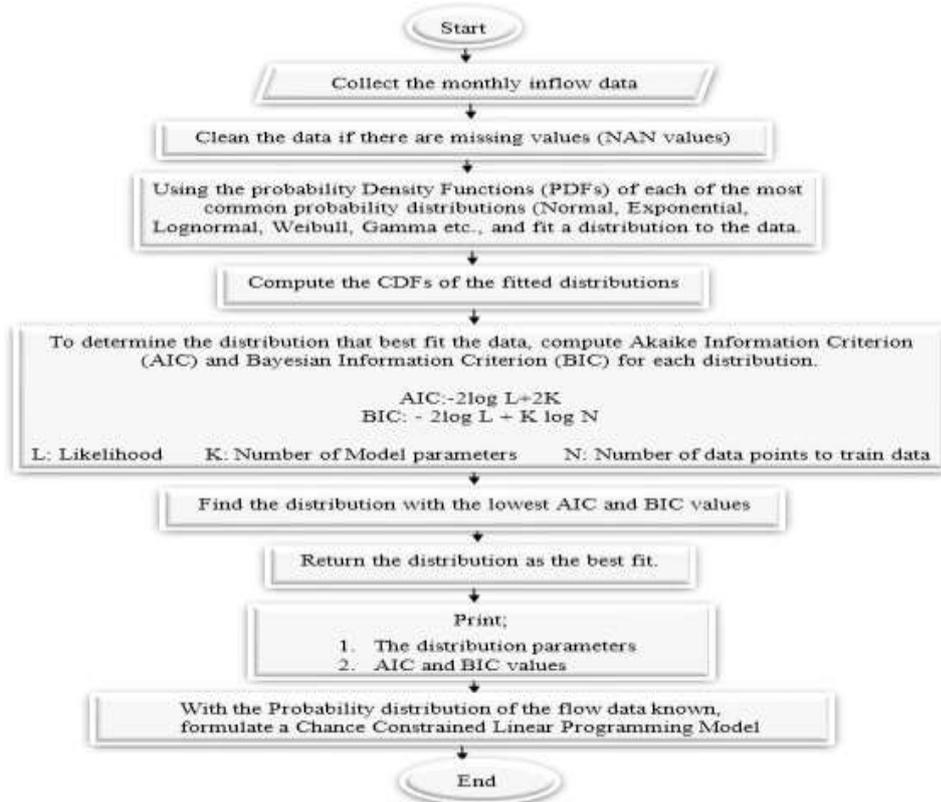


Fig 3: Flow Chart Probability Distributions Fitting to the Outflow Data

The inflow to a reservoir is the most important random variable that introduces uncertainty in reservoir planning and operation problems. In this work, a stochastic linear programming (LP), chanced constrained linear programming (CCLP) model was formulated to obtain the reservoir yield to meet a specified demand,  $D_t$ , during period  $t$ , at a specified level of reliability. Because the inflow is a random variable, a probability distribution is first fitted to the monthly inflow data. The MATLAB code was used for fitting the probability distribution to the data, of the cumulative distribution function (CDF) of the inflow discharge ( $F_{Q_t^{-1}}(\alpha)$ ) and the maximum yield ( $R_{ma}$ ) and minimum yield ( $R_{min}$ ). Where  $F_{Q_t^{-1}}(\alpha)$  denotes the flow,  $Q_t$ , at which the CDF value is  $\alpha$ , computed is at the reliability levels of 25%, 50%, 75%, and 85.27% . The goal then is to find maximum annual yield such that the monthly reservoir yield is able to meet demand fully

$$f(x_i; \alpha, \beta) = \left(\frac{\beta}{\alpha}\right) \left(\frac{x_i}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{x_i}{\alpha}\right)^\beta\right]$$

The Cumulative Distribution Function is given by,

$$F(x_i; \alpha, \beta) = 1 - \exp\left[-\left(\frac{x_i}{\alpha}\right)^\beta\right] \tag{2}$$

Where  $x_i$  is the  $i$ th value of the random variable  $X$ .

The likelihood function is given by,

$$L = \prod_{i=1}^n f(x_i, \hat{\theta})$$

The MLE of  $\hat{\theta}$  is the value  $\hat{\theta}$  that maximizes the likelihood function or the log-likelihood function. That is,

$$\frac{d \log L}{d \hat{\theta}} = 0$$

By substituting equation 1 into equation 3

$$L(x_i; \alpha, \beta) = \prod_{i=1}^n \left(\frac{\beta}{\alpha}\right) \left(\frac{x_i}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{x_i}{\alpha}\right)^\beta\right] \tag{5}$$

By taking the natural logarithms (ln) of both sides of Eqn. (5), this results to,

$$\ln L(x_i; \alpha, \beta) = n \ln \beta - n \beta \ln \alpha - \frac{1}{\alpha^\beta} \sum_{i=1}^n x_i^\beta + (\beta - 1) \sum_{i=1}^n \ln x_i \tag{6}$$

Now differentiating with respect to  $\alpha$  and  $\beta$  and equating to zero, this results into Equation 7 and Equation 8,

$$\frac{\partial \ln L(\alpha, \beta)}{\partial \alpha} = -\frac{n\beta}{\alpha} + \frac{\beta}{\alpha^{\beta+1}} \sum_{i=1}^n x_i^\beta = 0 \tag{7}$$

$$\frac{\partial \ln L(\alpha, \beta)}{\partial \beta} = \frac{n}{\beta} - n \ln \alpha - \frac{\sum_{i=1}^n x_i^\beta - \ln \alpha \sum_{i=1}^n x_i^\beta}{\alpha^\beta} + \sum_{i=1}^n \ln x_i = 0 \tag{8}$$

By solving equations 7 and 8 through elimination method and simplifying,

at all time period with certain specified level of reliability (assurance)

The method of Maximum Likelihood Estimation (MLE) is a common procedure to estimate parameters of a model's distribution which are assumed to be independent and identically distributed. The parameters are estimated by maximizing the likelihood function. In this study, six PDF (Normal, Gamma, Weibull, Exponential, Inverse Gaussian, and Lognormal) were fitted to the inflow data and the best distribution model based on the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) values was picked. The parameters of the six distributions were obtained or estimated using the Maximum Likelihood Estimation (MLE). The probability density function of a two parameter of Weibull distribution with scale parameter,  $\alpha > 0$  and shape parameter,  $\beta > 0$ , is given by,

$$\hat{\alpha} = \left(\frac{1}{n} \sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}} \tag{10}$$

$$\frac{1}{\beta} - \frac{\sum_{i=1}^n x_i^\beta \ln x_i}{\sum_{i=1}^n x_i^\beta} + \frac{1}{n} \sum_{i=1}^n \ln x_i = 0$$

The estimate  $\hat{\beta}$  must be solved numerically from equation 10 by methods such as the Newton-Raphson method or other root-finding algorithms can be employed to find  $\beta$ .

By defining equation 10 as,

$$f(\beta) = \frac{1}{\beta} - \frac{\sum_{i=1}^n x_i^\beta \ln x_i}{\sum_{i=1}^n x_i^\beta} + \frac{1}{n} \sum_{i=1}^n \ln x_i \tag{11}$$

The value of  $\beta$  such that  $f(\beta) = 0$  can be obtained through a combination of iterative and update process as given by equation 12 ,

$$\beta_{new} = \beta_{old} - \frac{f(\beta_{old})}{f'(\beta_{old})} \tag{12}$$

Where  $f'(\beta_{old})$  is the first derivative of Equation 11 with respect to parameter  $\beta$ . Starting with an initial guess of  $\beta_o$ , both the iteration and update processes must be carried out continuously until convergence as given by equation 13,

$$|\beta_{new} - \beta_{old}| \leq 0.000001 \tag{13}$$

The final estimate  $\hat{\beta}$  (shape parameter) thus obtained will then be substituted into equation 9 to obtain the scale parameter  $\hat{\alpha}$ . A MATLAB function was used for estimating these parameters.

Weibull distribution was fitted to the inflow data of each month and the result are as shown in table 1 below.

Model Selection Criterion

To compare the performance of the probability distribution functions, fitted on the inflow data, AIC and BIC approaches was employed. The AIC is given by,

$$\begin{aligned}
 & \text{AIC} \\
 & = -2 \log L(\hat{\theta}) \\
 & + 2k \qquad \qquad \qquad 14
 \end{aligned}$$

Where  $L(\hat{\theta})$  is the likelihood function of the data when evaluated at the maximum likelihood estimate of  $\theta$  and  $k$  is the number of estimated parameters? The BIC is given by,

$$\begin{aligned}
 & \text{BIC} \\
 & = -2 \log L(\hat{\theta}) \\
 & + k \log n \qquad \qquad \qquad 15
 \end{aligned}$$

**Chance Constrained LP Model**

The deterministic maximum reservoir yield problem can be written as:

$$\begin{aligned}
 & \text{Max } \sum_{t=1}^{12} R_t \\
 & \text{Subject to:} \\
 & S_t + Q_t - R_t - E_t = S_{t+1} \quad \text{for all } t \\
 & R_t \geq D_t \qquad \qquad \qquad \text{for all } t \\
 & R_t \leq R_t^{\text{max}} \qquad \qquad \qquad \text{for all } t \\
 & S_t \leq K \qquad \qquad \qquad \text{for all } t \\
 & S_t \geq S_{\text{min}} \qquad \qquad \qquad \text{for all } t \\
 & K \geq 0 \\
 & R_t \geq 0; S_t \geq 0 \qquad \qquad \qquad \text{for all } t
 \end{aligned}$$

Where  $K$  is the reservoir capacity.  
 $S_t$  is the storage at the beginning of time period  $t$ .  
 $Q_t$  is the inflow during time period  $t$ .  
 $R_t$  is the release/yield during time period  $t$ .  
 $R_t^{\text{max}}$  is the maximum release that can be made in period  $t$ .  
 $D_t$  is the demand to be met in period  $t$ .  
 $S_{\text{min}}$  is the minimum storage below which no release is made.  
 In this model, reservoir demands are satisfied 100% of the time, assuming a feasible solution and deterministic variables. However, the inflow  $Q_t$  is a random variable and To apply constraints (Equation 2) to Equation 4) within an optimization algorithm, we need to determine the probability distributions of  $R_t$  and  $S_t$  based on the known distribution of  $Q_t$ . However, due to the interdependence of  $S_t$ ,  $Q_t$ , and  $R_t$  through the continuity equation, deriving the probability

**Linear Decision Rule (LDR)**

The simplest form of LDR is defined as follows:

$$R_t = S_t + Q_t - b_t \qquad \qquad \qquad 20$$

$b_t$  is a deterministic parameter called the decision parameter. Equation 5 assumed that all the available amount of water is being considered while making release decision. Depending on the amount of water considered in making release decision, several linear decision rules could be written as follows:

$$R_t = S_t + \beta Q_t - b_t \qquad \qquad \qquad 0 \leq \beta \leq 1 \qquad \qquad \qquad 21$$

In this study, a conservative release policy in which the value of  $\beta$  is 1 (i.e. all of the available water is used in making release decision) is considered.  
 note that the storage continuity equation is

thus introduces uncertainty in the problem. Its probability distribution can be estimated using historical inflow data. Since storage  $S_t$  and release  $R_t$  depend on the random variable  $Q_t$ , they are also considered random variables.

To solve the problem, the deterministic model above was formulated as a Chance Constraint Linear Programming model as follows:

$$\begin{aligned}
 & \text{Max } \sum_{t=1}^{12} R_t \\
 & \text{Subject to:} \\
 & P[R_t \geq D_t] \geq \alpha_1 \qquad \qquad \qquad \text{for all } t \\
 & P[R_t \leq R_t^{\text{max}}] \geq \alpha_2 \qquad \qquad \qquad \text{for all } t \\
 & P[S_t \leq K] \geq \alpha_3 \qquad \qquad \qquad \text{for all } t \\
 & P[S_t \geq S_{\text{min}}] \geq \alpha_4 \qquad \qquad \qquad \text{for all } t \\
 & b_t \geq 0 \qquad \qquad \qquad \text{for all } t \\
 & K \geq 0
 \end{aligned}$$

First the constraint relating the release,  $R_t$  (random) and demand,  $D_t$  (deterministic), is expressed as a chance constraint as follows:

$$P[R_t \geq D_t] \geq \alpha_1 \qquad \qquad \qquad 16$$

That is, probability of release equaling or exceeding the known demand is at least equal to  $\alpha_1$  (reliability level). This can also be interpreted as the reliability of meeting the demand in period  $t$  is at least  $\alpha_1$ .

Similarly, the maximum release and the maximum and minimum storage constraints are written as:

$$P[R_t \leq R_t^{\text{max}}] \geq \alpha_2 \qquad \qquad \qquad 17$$

$$P[S_t \leq K] \geq \alpha_3 \qquad \qquad \qquad 18$$

$$P[S_t \geq S_{\text{min}}] \geq \alpha_4 \qquad \qquad \qquad 19$$

distributions of both  $S_t$  and  $R_t$  is generally not feasible. To address this challenge and facilitate the use of linear programming, an appropriate linear decision rule is defined as follows.

$$S_{t+1} = S_t + Q_t - R_t \qquad \qquad \qquad 22$$

Therefore, from equation 5 and 7,  $S_{t+1} = b_t$

**Deterministic equivalent of the CCLP**

The deterministic equivalent of the Chance Constrained LP model above can be expressed as follows:

$$\begin{aligned}
 & \text{Min } \sum_{t=1}^{12} R_t \\
 & \text{subject to:} \\
 & D_t + b_t - b_{t-1} \leq F_{Q_t}^{-1}(1 - \alpha_1) \qquad \qquad \qquad \text{for all } t \\
 & R_t^{\text{max}} + b_t - b_{t-1} \geq F_{Q_t}^{-1}(\alpha_2) \qquad \qquad \qquad \text{for all } t \\
 & b_{t-1} \leq K \qquad \qquad \qquad \text{for all } t \\
 & b_{t-1} \geq S_{\text{min}} \qquad \qquad \qquad \text{for all } t \\
 & b_t \geq 0 \qquad \qquad \qquad \text{for all } t \\
 & K \geq 0
 \end{aligned}$$

**Putting in all the values the final model becomes:**

$$\begin{aligned}
 & \text{Max } \sum_{t=1}^{12} R_t \\
 & \text{subject to:}
 \end{aligned}$$

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$$\begin{aligned}
 b_t - b_{t-1} &\leq F_{Qt}^{-1}(1 - \alpha_1) - D_t && \text{for all } t \\
 b_t - b_{t-1} &\geq F_{Qt}^{-1}(\alpha_2) - R_t^{\max} && \text{for all } t \\
 b_{t-1} &\leq K && \text{for all } t \\
 b_{t-1} &\geq S_{\min} && \text{for all } t \\
 b_t &\geq 0 && \text{for all } t
 \end{aligned}$$

Where K is the reservoir capacity.

$F_{Qt}^{-1}(1 - \alpha_1)$  is the flow, q at which the CDF value is  $1 - \alpha_1$ .  
 $R_t^{\max}$  is the maximum release that can be made in period t.  
 $D_t$  is the demand to be met in period t.  
 $S_{\min}$  is the minimum storage below which no release is made.  
 $b_{t-1}$  is storage at the next time period t.

3.0 RESULTS AND DISCUSSIONS

Monthly Inflow Probability Distribution

In order to determine the best probability distribution model to adopt, reservoir inflow data were put under probability distribution analysis, amongst which were normal, gamma, lognormal, exponential, weibull and inverse Gaussian. The

best fit model was weibull. This is because it had the least value for AIC & BIC. Below is the inflow probability distribution for the month of January to December. The probability distribution with smaller values of AIC and BIC are thus selected.

Table 1: Inflow Data at 25<sup>th</sup>, 50<sup>th</sup>, 75<sup>th</sup> and 85.27<sup>th</sup> Percentile and Reservoir Characteristics

PERIOD	$F_{Qt}^{-1}(\alpha_1) = F_{Qt}^{-1}(0.25)^{**}$ (MCM*)	$F_{Qt}^{-1}(\alpha_2) = F_{Qt}^{-1}(0.50)^{**}$ (MCM*)	$F_{Qt}^{-1}(\alpha_3) = F_{Qt}^{-1}(0.75)^{**}$ (MCM*)	$F_{Qt}^{-1}(\alpha_4) = F_{Qt}^{-1}(0.8527)^{**}$ (MCM*)	K (MCM*)	$R_{\max}$ (MCM*)	$S_{\min}$ (MCM*)	D (MCM*)
JANUARY	0.8279	1.7267	3.082	4.0381	186.1	129.6	18.99	15.9
DEBRUARY	0.3928	0.9481	1.8987	2.6249	186.1	129.6	18.99	16.9
MARCH	0.414	0.7361	1.1584	1.4312	186.1	129.6	18.99	16.9
APRIL	0.5148	1.1308	2.1027	2.8079	186.1	129.6	18.99	16.5
MAY	3.3219	6.1036	9.8589	12.3293	186.1	129.6	18.99	14.3
JUNE	8.9934	17.2549	28.8382	36.6425	186.1	129.6	18.99	10.6
JULY	37.042	50.918	65.4313	73.5486	186.1	129.6	18.99	3.6
AUGUST	90.7692	117.3779	143.7438	157.9891	186.1	129.6	18.99	2.8
SEPTEMBER	142.579	182.6475	222.0205	243.1799	186.1	129.6	18.99	7.1
OCTOBER	38.7526	59.7829	84.1355	98.6695	186.1	129.6	18.99	10.5
NOVEMBER	8.3681	12.7749	17.8311	20.8311	186.1	129.6	18.99	14.8
DECEMBER	2.2909	4.0457	6.334	7.8066	186.1	129.6	18.99	15.7

\*MCM:- Mega Cubic Meter  
 \*\* Flow at specified cdf

Table1 gives the  $F_{Qt}^{-1}(\alpha)$  value for the inflows at the reliability levels of 25%, 50%, 75%, and 85.27% earlier chosen is presented in Table 1.  $R_{\max}$  is the maximum monthly water release,  $S_{\min}$  is the minimum storage or dead storage of

the reservoir and K is the reservoir capacity of Galma dam. The Weibull distribution with scale factor A of 25.3522 and shape factor B of 0.551905 best fit the inflow data.

The Monthly Inflow at Specified Reliability Levels (Figure 4)

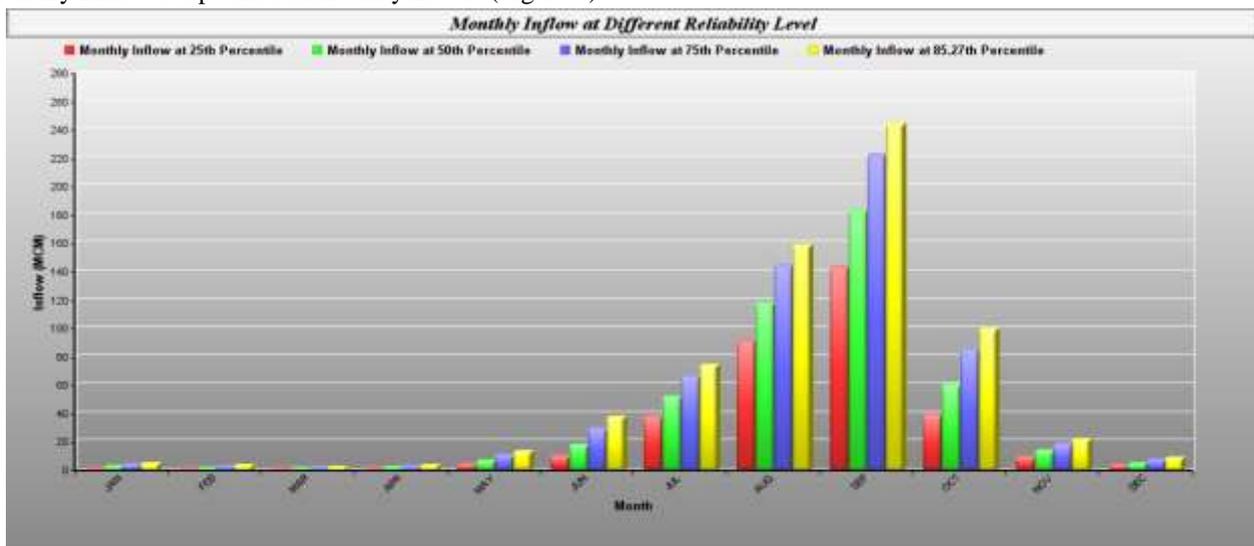


Figure 4: Inflows at different reliability levels

By computing the inverse Cumulative Density Function at specified probabilities (reliabilities) for Weibull Distribution, the inflows at those probabilities were obtained and plotted as

in Figure 14 above. The probability of meeting the water demand to the different parameters indicates that the water requirement will meet the demands in the month of August

and September and low probability of meeting demand in April.

The optimized Reservoir Yield at Different Reliability Levels (Figure 5)

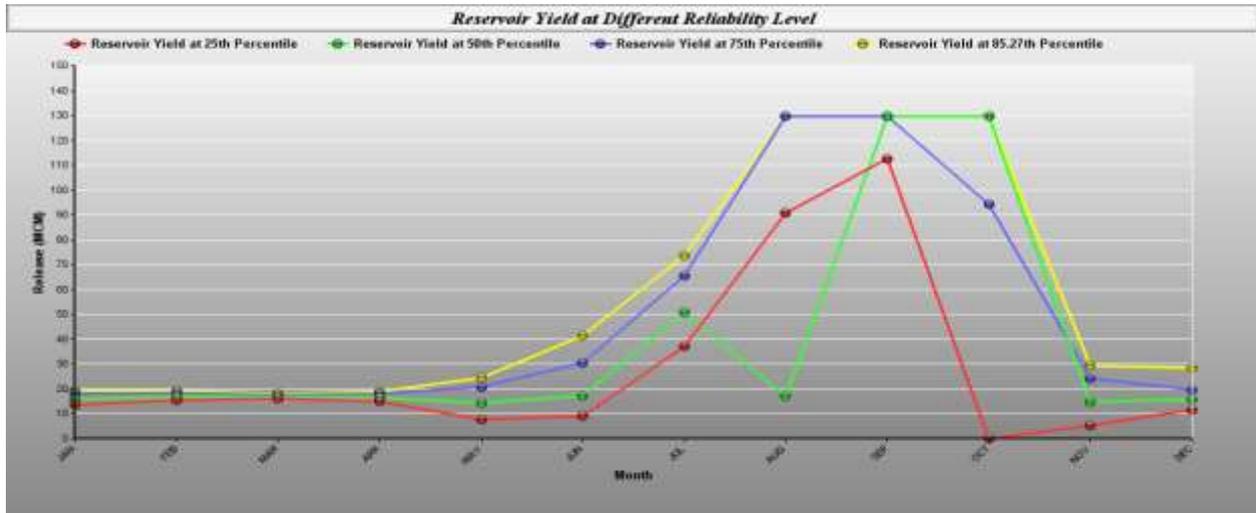


Figure 5: Reservoir yields at different reliability levels

Figure 5 is the LINGO plot of monthly release decisions at different reliability levels. The most dependable release decision is the one occurring at 85<sup>th</sup> percentile or 0.8527 reliability level. This is because at this level of reliability, Reservoir Storage at Different Reliability Levels (Figure 6)

demand is always satisfied. At this level, the reservoir will meet the water demand at 85% capacity while it will not meet the demand at 20% capacity

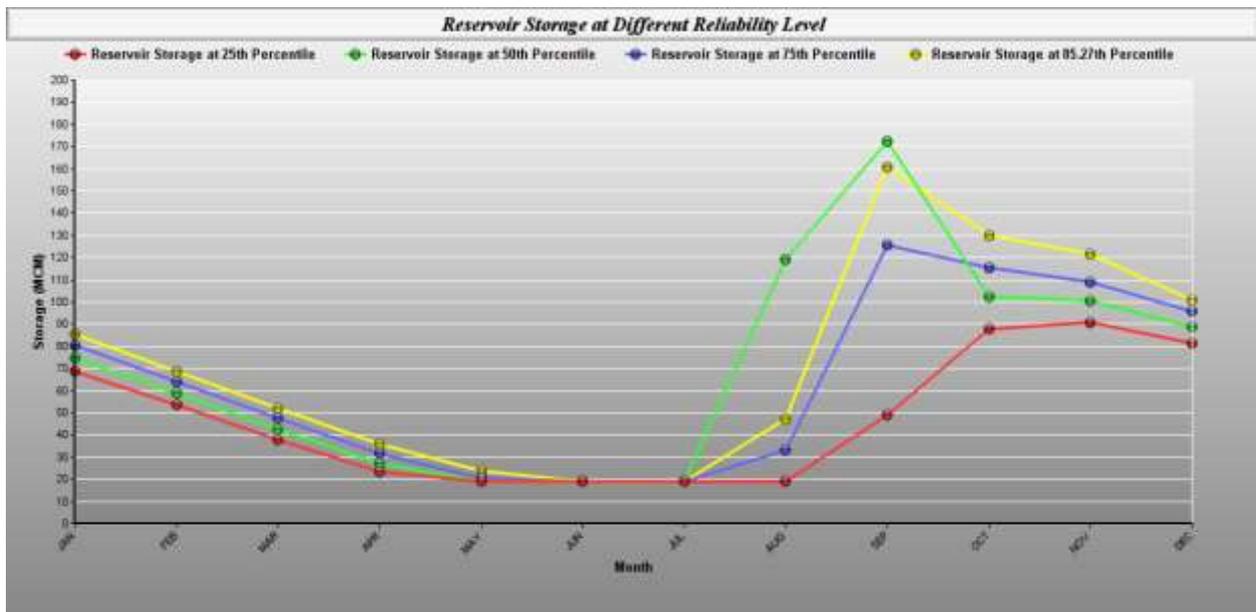


Figure 6: Storage at different reliability levels

Figure 6 is the LINGO plot monthly storage decisions. The plot is showing that from December to May when the storage is dropping, the release is also kept low as much as possible. Also from around June to September when the storage spikes,

the release also increases. This behavior shown by the model is indeed correct. This is because it is only when the reservoir has much water in storage, that the yield is expected to spike. Reservoir Yield – Reliability Level Plot (Figure 7)

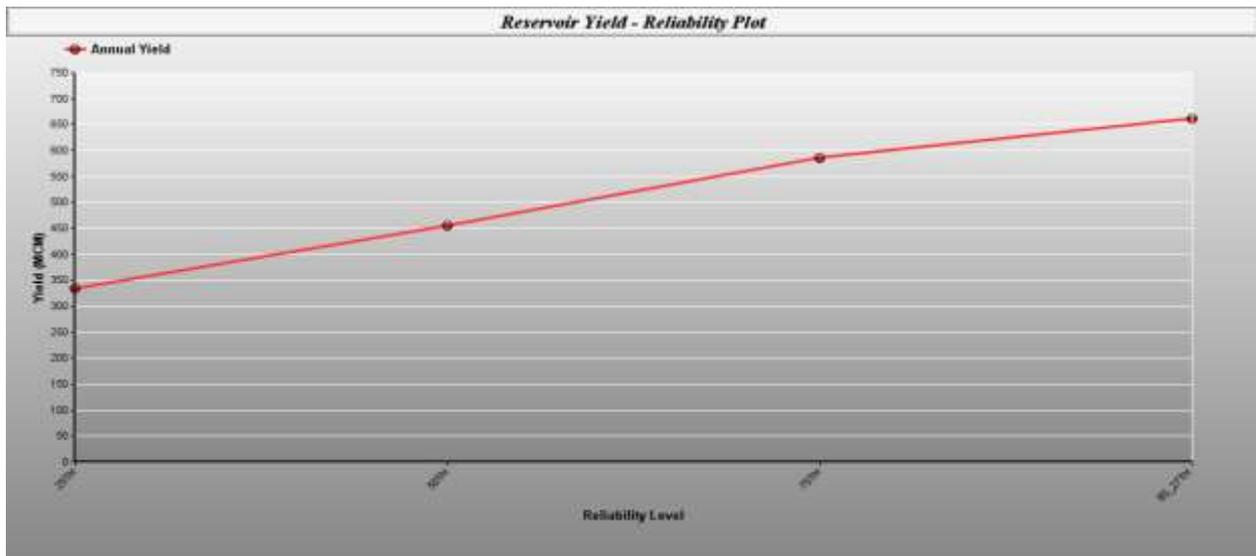


Figure 7: Plot of annual yield against reliability levels

Figure 7 is the LINGO plot showing total annual yield at different levels of reliability, the maximum yield that can be realized annually is 669.8987 MCM. This occurs at 0.8527 reliability level. Beyond this level, the model becomes infeasible. That is, at higher reliability level, no more water will be available to satisfy or meet up with the demand at all time. Reservoir yield at different reliability level, with the maximum yield of 661.8987 (MCM) at 85.27% reliability

level and the minimum yield of 334.2666 (MCM) at 25% reliability level, at this level, there is the chance of not meeting the water demand for 25% of the supply time (Months). This is necessary as it is used to reassess water demand that can be satisfied by the reservoir. The overall volume or storage is dependent on the total demand, reliability and the catchment supplying water to the reservoir.

Table 2: Monthly Release Decisions at Different Reliability Level

Month	Release MCM @ (25th Percentile)	Release MCM @ (50th Percentile)	Release MCM @ (75th Percentile)	Release MCM @ (85th Percentile)
Jan	13.6459	15.9000	18.1541	19.4328
Feb	15.3941	16.9000	18.4059	19.3076
Mar	16.1556	16.9000	17.6444	18.0499
Apr	14.9121	16.5000	18.0879	19.0045
May	7.7630	14.3000	20.8370	24.4218
Jun	8.9934	17.2549	30.4448	41.4373
Jul	37.0420	50.9180	65.4313	73.5486
Aug	90.7692	17.0742	129.6000	129.6000
Sep	112.5974	129.6000	129.6000	129.6000
Oct	0.0000	129.6000	94.2236	129.6000
Nov	5.3370	14.8000	24.2630	29.3332
Dec	11.6569	15.7000	19.7431	28.5630
<b>Annual Yield (MCM)</b>	<b>334.2666</b>	<b>455.4471</b>	<b>586.4351</b>	<b>661.8987</b>

Table 2 indicates the total volume of water (MCM) that can be released at different reliability level for each month. The month in which the highest volume of water can be released in the 25<sup>th</sup> percentile reliability level is September, with 112.5974MCM. In the 50<sup>th</sup> percentile reliability level, the months of September and October have the highest volume of release (129.6MCM). Also in the 75<sup>th</sup> percentile reliability

level, August and September have 129.6MCM separately as the highest volume of released water. While in the 85<sup>th</sup> percentile reliability level, the months of August, September, October each have a total volume of 129.6MCM that can be released, which is the highest for this level. The above table will therefore guide in the water release policy for Galma dam.

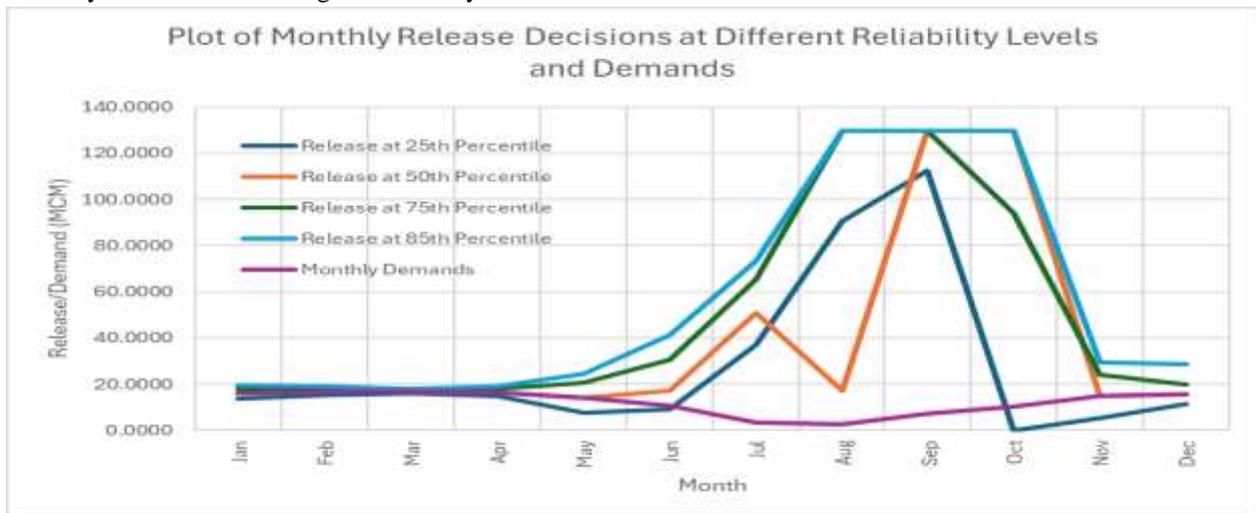
**Table 3: Monthly Storage Decisions at Different Reliability Level**

Month	Storage MCM @ (25th Percentile)	Storage MCM @ (50th Percentile)	Storage MCM @ (75th Percentile)	Storage MCM @ (85th Percentile)
Jan	68.5713	74.6714	80.5531	85.3753
Feb	53.5700	58.7195	64.0459	68.6926
Mar	37.8284	42.5556	47.5599	52.0739
Apr	23.4311	27.1864	31.5747	35.8773
May	18.9900	18.9900	20.5966	23.7848
Jun	18.9900	18.9900	18.9900	18.9900
Jul	18.9900	18.9900	18.9900	18.9900
Aug	18.9900	119.2937	33.1338	47.3791
Sep	48.9716	172.3412	125.5543	160.9590
Oct	87.7242	102.5241	115.4662	130.0285
Nov	90.7553	100.4990	109.0343	121.5264
Dec	81.3893	88.8447	95.6252	100.7700

The above table shows that the dead storage of Galma dam is 18.99MCM occurring in the months of June and July. It also

reveals the volume of water stored in every month of the year in the 25<sup>th</sup>, 50<sup>th</sup>, 75<sup>th</sup> and 85<sup>th</sup> percentile reliability level of the reservoir.

Plot of Monthly Release Decisions against Monthly Demands



**Figure 8: Plot of Release/Demand against month**

From the plot above, it is obvious that the most dependable yield is the one at the maximum reliability level of 0.8527 (85.27%). At this level of reliability (assurance), the release fully satisfied demand always. This is the maximum reliability level (level of assurance) beyond which the

Solution becomes infeasible. The infeasibility means there is not much inflow to fully meet demand always and therefore the constraint that release will be equaled or exceed demand will be violated at a higher reliability than the maximum of 85.27%.

**CONCLUSION**

This study presents a reliability analysis framework for optimizing the operations of Galma’s dam reservoir using an explicit stochastic modeling approach. The methodology incorporates the inherent uncertainties in hydrological inflows, water demands and other operating conditions through a stochastic optimization model. The stochastic optimization model is formulated as a chance-constrained programming problem, which aims to maximize the long-term reliability of meeting various operational objectives, such as water supply, flood control subject to various

physical, operational and environmental constraints. The model uses an explicit representation of the stochastic nature of the system variables, including inflows, water demands and other operating conditions, to quantify the probability distributions of the system’s performance indicators. The reliability analysis is conducted by evaluating the probability of satisfying the operational constraint under different reliability levels. This shows for the identification of optimal operating policies that balance the tradeoffs between competing objectives and ensure a desired level of system reliability. The proposed framework is applied to the Galma’s

dam reservoir system, and the results demonstrate the effectiveness of the stochastic optimization approach in enhancing the reliability and resilience of the reservoir operations. The findings of this study provide valuable

insights for water resources managers and decision-makers in developing robust and sustainable reservoir management strategies, particularly in the face of increasing uncertainties due to climate change and other environmental factors.

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