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Impedance Matching Analysis A Case of Simple 2-Particle Elastic Collision

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ABSTRACT: This study analyses the impedance matching (IM) of the 2-particles elastic collision phenomenon by considering factors such as particle velocity, particle mass, and 2-particles collision restitution coefficient. Results show that IM of 2-particle elastic collision is affected by 3 particle velocity ratios (v_2/v_1, v_2'/v_1, and v_1'/v_1), 1 particle mass ratio (m_2/m_1), and 1 collision restitution coefficient (e). Two particle collisions with maximum IMs condition or the kinetic energy portion transferred 100% only occurs if the particle mass of causing the collision is equal to the particle mass of the target ($m_2/m_1 = 1$), the 2-particles collision condition is the perfectly elastic (e=1), and the particle-1 velocity of between in before and after the collision is the same ($v_1'/v_1 = 1$). Action of controlling the particle-1 velocity of before the collision so that it can reach the critical value where the maximum IMs condition or the kinetic energy (KE) portion transferred in maximum amount, this can only be done by ensuring that the masses of the two particles are the same and that the collision of the two particles must be perfectly elastic.

KEYWORDS: Impedance matching, kinetic energy transfer, 2-particles elastic collision.

I. INTRODUCTION

Elastic collisions between 2-particles in Fig. 1 are an important physical phenomenon that occurs in many fields such as mechanical engineering and aeronautical engineering. This collision process involves the exchange of energy and momentum between particles, which affects the behaviour and dynamics as the whole particle system. One of key entrances in analysing the elastic collision phenomenon is the IM variable which plays an important role in determining the energy efficiency and performance of the system.

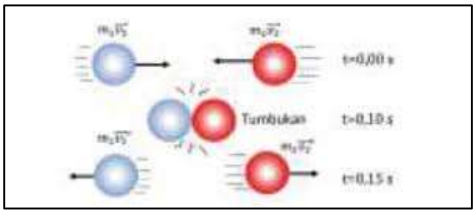

Fig.1 Elastic collision of 2 particles

Elastic collisions and IM have been a topic of intense research. Discussion of elastic collision provides a foundation for understanding the behaviours of mechanical systems [1, 2]. IM techniques and their applications play an important role in vibrational and wave systems. This IM technique shows that there is an inherent potential for the purpose of mechanical system optimization [3]. Apart from that, there is a research state the open application opportunities in various fields, such as the automotive and aerospace industries [4].

IM is a technique used to know a physical phenomenon state where the energy amount transferred between two systems with different impedances is maximum [5, 6, 7]. In the elastic collision's context, IM allows setting particle parameters to achieve optimal energy efficiency.

From the explanation above, it shows that the IM study of the 2-particles elastic collision phenomenon is interesting to do. The study will consider influential factors such as particle velocity, particle mass, and restitution coefficient. This study will broaden the understanding of IM in mechanical systems and improve their energy efficiency. It will also help in understanding the basic principles of IM and its applications in various fields.

II. METHODOLOGY

This research uses an analytical and theoretical descriptive method based on mathematical models and curves from IM of the 2-particles elastic collision phenomenon. There are 4 stages of work carried out to complete this research.

Preparation stage includes: (a) Literature study, namely reviewing the theory of elastic collision, IM, and its applications; (b) Definition of variables, namely determining particle parameters (velocity, mass, restitution coefficient); and (c) Formulation of mathematical models, namely developing equations of motion from Newton's law.

Theoretical analysis stage includes: (a) IM analysis, namely analysing the concept of IM in the simple 2-particles elastic collisions; (b) Development of mathematical models, namely developing mathematical models that predict energy efficiency in the form of optimally transferred translational KE; and (c) Sensitivity analysis, namely analysing the particle parameters effect on energy efficiency or optimally transferred translational KE conditions.

Curve study stage includes: (a) Selection of a software program, namely using the Microsoft Excel software program; (b) Development of curve variables, namely developing special quantities that play a role in describing the simple 2-particles elastic collision phenomenon curve related specifically to their IM; and (c) Validation of results, namely comparing the curves tendency obtained in relation to the results of theoretical analysis that have been obtained.

Testing and analysis stage includes: (a) Model testing, namely testing the IM mathematical model and IM curve; (b) Analysis of results, namely analysing the IM curve tendency results and comparing it's with the theoretical analysis results that have been obtained; and (c) The effect of parameters, namely analysing the particle parameters effect on energy efficiency or the amount of optimally transferred translational KEs.

III.PARTICLE VELOCITY EFFECT ON IM

Elastic collisions of 2-particles include straight (translational) motion phenomena that obey Newton's second law with a mathematical statement of the form:

$$\vec{F} = m\vec{a} \ldots (1)$$

Where $\vec{F}$ is the resultant force vector acting on the 2-particles system, or particle-1, or particle-2; m is the total mass of the 2-particles system, or particle-1, or particle-2; and $\vec{a}$ is the acceleration vector of the 2-particles system, or particle-1, or particle-2.

Kinematics studies show that the particle system linear acceleration is change in particle system velocity per unit time. In other words, acceleration $(\vec{a})$ is ratio of between the final velocity $(\vec{v})$ minus initial velocity $(\vec{v}_0)$ against with time interval (Δt) or $\vec{v} - \vec{v}_0 = \vec{a}\Delta t$. Thus, mathematical statement (1) can be written as:

$$\vec{F}\Delta t = m\vec{v} - m\vec{v}_0 \ldots (2)$$

For the elastic collision case of 2-particles, each with mass m_1 and m_2; each with velocities $\vec{v}_1$ and $\vec{v}_2$ for before collision, $\vec{v}_1{}'$ and $\vec{v}_2{}'$ for after collision, the mathematical statement (2) can be written as:

$$(\vec{F}_{12} + \vec{F}_{21})\Delta t = (m_2\vec{v}_2' + m_1\vec{v}_1') - (m_2\vec{v}_2 + m_1\vec{v}_1) \ldots (3)$$

Where $\vec{F}_{12}$ is the instantaneous push force vector by particle-1 to particle-2 and $\vec{F}_{21}$ is the instantaneous push force vector by particle-2 to particle-1. Naturally, in every of collision phenomenon type, third Newton's law applies where the action force vector and the reaction force vector are always equal in magnitude and opposite in direction or $\vec{F}_{12} = -\vec{F}_{21}$. Consequently, the mathematical statement (3) can be written as:

$$m_2\vec{v}_2' + m_1\vec{v}_1' = m_2\vec{v}_2 + m_1\vec{v}_1 \ldots (4)$$

IM analysis is always based on a logical condition that the energy amount transferred between particle-1 and particle-2 must take place efficiently. This means that the energy amount transferred in the system between the 2-particles must be maximum. The energy type transferred in the 2-particles elastic collision phenomenon is KE, with a mathematical statement form:

$$KE = \frac{1}{2}mv^2 \ldots (5)$$

Where $m$the particle is mass, v is the particle translational velocity. If the KE amount transferred by each particle in the system is KE_1, which is the particle-1 KE before collision and $KE_2{}'$, which is the KE received by particle-2 after collision, the mathematical statement for each of these quantities type can be written in form:

$$EK_1 = \frac{1}{2}m_1{v_1}^2 \ldots (6)$$

$$EK_2{}' = \frac{1}{2}m_2\left({v_2'}^2 - {v_2}^2\right) \ldots (7)$$

Based on equations (6) and (7), the mathematical model as the basis for IM analysis is the efficiency concept of related activities. It can be expressed as the ratio between the KE received by particle-2 and the KE before the collision, which was initially possessed, and then transferred by particle-1. Therefore, through the help of mathematical statements (4), the IM mathematical model for the 2-particles elastic collision phenomenon, can be written in the form of:

$$\eta = \frac{EK_2{}'}{EK_1} = \left(\frac{v_2}{v_1} + \frac{v_2{}'}{v_1}\right)\left(1 - \frac{v_1{}'}{v_1}\right) \ldots (8)$$

From the mathematical statement (8), it appears that the IM of a 2-particles linear collision is affected by three different types of particle velocity ratios, namely (v_2/v_1), $(v_2{}'/v_1)$, and $(v_1{}'/v_1)$. These ratios affect the system IM through different means. For the ratios (v_2/v_1) and $(v_2{}'/v_1)$, they have an impact in the same value change against IM value change direction. For the ratio $(v_1{}'/v_1)$, it has an impact in the opposite value change against IM value change direction. This means that if the ratio values of (v_2/v_1) and $(v_2{}'/v_1)$ are increasing but the ratio $(v_1{}'/v_1)$ is getting bigger of closser 1, the KE transferred from particle-1 to particle-2 will getting smaller of closser zero. The same applies to the opposite condition.

For the case of a simple 2-particle elastic collision where the initially particle-2 is at rest so v_2 = zero applies. Based on the IM mathematical model in equation (8), it is verified that there are only two types of particle velocity ratios. Each of: (a) The (v_2'/v_1) velocity ratio has the impact of enlarge or positive on IM, especially for $(v_2'/v_1) > 1$ and vice versa for $0 < (v_2'/v_1) < 1$. (b) The (v_1'/v_1) velocity ratio impact always weakens or negative impact on IM because the value always satisfies $0 \leq (v_1'/v_1) \leq 1$.

Referring to the work-energy principle, it can be stated that the net work in the 2-particles perfect elastic collision phenomenon of is 0. This happens because the resultant of force vector acting in the system is zero. The 2 force vectors acting in the system are equal in magnitude but opposite in direction. The sum of the particle system translational KE, just before and after the collision is the same. The mathematical statement of this condition can be written as:

$$\frac{1}{2}m_1 {v_1'}^2 + \frac{1}{2}m_2 {v_2'}^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \ldots (9)$$

Based on equation (9), the IM mathematical model of 2-particles perfect elastic collision from equation (8) can be changed and rewritten in the form of:

$$\eta = \frac{EK_2'}{EK_1} = \left(1 - \left(\frac{v_1'}{v_1}\right)^2\right) \ldots (10)$$

The (v_1'/v_1) velocity ratio character as verified by equation (8), appears to be reverified by equation (10). However, this system IM mathematical model appears only to be affected by square of velocity ratio of particle 1 after with before collision or $(v_1'/v_1)^2$. This means that if this particle-1 velocity ratio is change to 50% of initially condition, the KE transfer from particle-1 to particle-2 change to 75% of initially condition. The lower the particle-1 velocity after the collision, the higher the transfer KE to particle-2. When the particle-1 condition after collision stopped, the transfer KE portion is 100%. The same applies for the opposite condition.

It appears explicitly from equation (10) for $v_1' \neq 0$ that for a perfectly elastic collision type of 2-particles, it is not guaranteed to produce a condition where η = 1 even though the sum of the KE of the system, immediately after and before collision is same. The KE transferred by particle-1 and received by particle-2 is not guaranteed to be 100%.

A deeper IM study of 2-particle perfect elastic collisions can be carried out through the curve representation of the dependent variable η as a function of the independent variable (v_1'/v_1) as expressed by equation (10). This curve representation is shown in Figure 2.

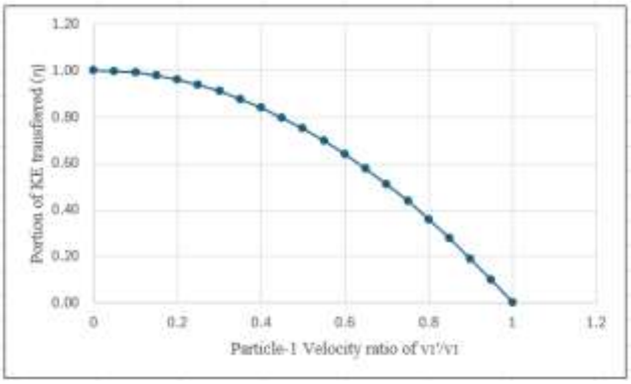

Fig. 2 Dependence of η on v_1'/v_1 for a 2-particles perfectly elastic collision

From Figure 2, IM curve shape as a function of (v_1'/v_1) is a half-parabola with a highest point at (0, 1) and two intersections point with the horizontal axis at (-1, 0) and (1, 0). This means that the highest efficiency of the IM mathematical model or KE that transfered of maximum occur if the particle-1velocity, after the collision is zero or the particle-1 condition, immediately after the collision is stop.

IV. PARTICLE MASS EFFECT ON IM

For the simple case studied, namely a perfect elastic collision where the initial condition of particle-2 is at rest, then mathematical operations with the help of equations (4) and (9) can obtain mathematical expressions of the velocity of particle-1 and particle-2 after the collision, each in the form:

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{1 - \mu_{21}}{1 + \mu_{21}} v_1 \ldots (11)$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 = \frac{2}{1 + \mu_{21}} v_1 \ldots (12)$$

Where $\mu_{21} = m_2/m_1$.

Based on equation (11) it can be verified that the particle velocity ratio between after and before the collision depends on the ratio between the 2-particles mass difference and the total mass of the particles involved. The difference in the 2 particles mass is getting smaller, the velocity ratio of v_1'/v_1 is getting smaller and closer to zero so the KE conditions being transferred are getting higher, as can be seen in Figure 2. For special conditions where $\mu_{21} = 1$ where the 2-particles masses are the same, the particle-1 velocity after the collision is 0 or shortly after the collision, particle-1 condition immediately stops.

Based on equation (12), it can also be verified that the particle-2 velocity ratio of after the collision with the particle-1 velocity before the collision depends on the mass ratio of particle-2 to particle-1. The greater of particle-2 mass (target) compared to the particle-1 mass so that μ_{21} is greater, than the particle-2 velocity after the collision slowdowns and maybe goes to zero. For special conditions of $\mu_{21} = 1$ where the 2-particles’ masses are same, the magnitude and direction of the particle-2 velocity after the collision are the same as the particle-1 velocity before the collision. The KE of particle-2 after the collision is the same as the KE particle-1 before the collision. This means that the KE received by particle-2 is 100% sourced from the KE of particle-1 before the collision.

From equations (11) and (12) then equation (10) as a mathematical equation model that underlies IM analysis, can then be changed and written in the form:

$$\eta = \frac{EK_2{}'}{EK_1} = \frac{4\mu_{21}}{(1+\mu_{21})^2} \ldots (13)$$

It is shown that the efficiency of transfer energy in a 2-particles perfect elastic collision depends explicitly and absolutely on the ratio of the particle-2 mass and the particle-1 mass or μ_{21}. This means that the transferred energy quality is highly dependent on μ_{21}.

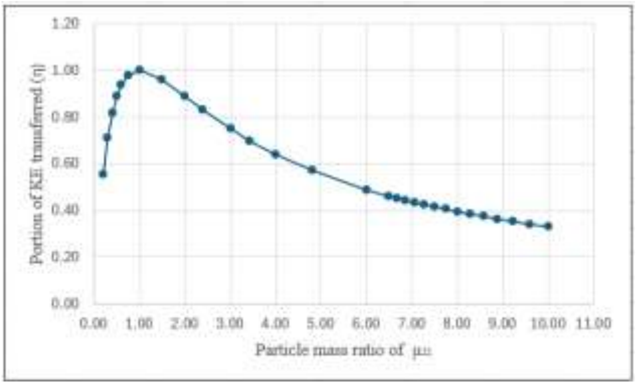

Fig. 3 Dependence of η on μ_{21} for a 2-particles perfect elastic collision

Next, three different particle mass ratio conditions were studied, namely $m_2/m_1 < 1$, $m_2/m_1 = 1$, dan $m_2/m_1 > 1$. In this ratio will be found the most efficient condition where the amount of KE transferred from particle-1 to particle-2 is maximum. Theoretically, this condition can be obtained by applying the calculus method, where this concept states that the dependent variable value against the independent variable is optimal (critical) if the differentiation of the dependent variable on the independent variable is zero. The mathematical statement for this case can be written in the form:

$$\frac{d\eta}{d\mu_{21}} = 0 \ldots (14)$$

Applying equation (14) with the IM function as stated by equation (13) then the optimum condition can be obtained only if μ_{21} = 1. This means that in a 2-particles perfectly elastic collision, the amount of KE transferred from particle-1 to particle-2 will be maximum, if the particle-2 mass is the same as the particle-1 mass. Analytically using the Microsoft Excel program, the results of this theoretical study can be displayed more sharply through analysis of the (13)-equation curve. The curve data is conditioned where the particle-2 mass fixed (600-grams) and the particle-1 mass to change, from 60-grams to 3000-grams. This condition curve is shown in Figure 3.

From Figure 3, when m_1 increases from 60-grams and then approaches m_2 = 600-grams, the amount of KE transferred increases and ultimately approaches the maximum value. The condition η = 1 occurs when $m_1 = m_2$. When m_1 is added, from 600-grams and until it exceeds the value of m_2 (600-grams), it appears that the amount of KE transferred decreases (η < 1) and towards a very small value or η approaches zero.

V. RESTITUTION COEFFICIENT EFFECT ON IM

The next study is the effect of changes in the restitution coefficient on the IM of mathematical model for each type of collision. For a 2-particles perfectly elastic collision, the system said have restitution coefficient = 1. The restitution coefficient is the ratio of the 2-particles relative velocity after the collision to the 2-particles relative velocity before the collision [8, 9, 10]. The mathematical expression for this restitution coefficient definition can written in the form:

$$e = \frac{v_2{}' - v_1{}'}{v_1 - v_2} \ldots (14)$$

In fact, this equation (14) for the restitution coefficient above can formulated through algebraic operations by utilizing the momentum and KE conservation law in translational motion in with the assumption that the system KE after the collision ≤ KE of system before the collision. For collisions where the initial particle-2 condition is rest, from equation (4) and (14), an IM equation model which explains the effect of changes in the restitution coefficient on the amount of KE transferred from particle-1 to particle-2 can formulated. Mathematical expression of IM for this case can written in the form:

$$\eta = \frac{EK_2{}'}{EK_1} = \frac{(1+e)^2\mu_{21}}{(1+\mu_{21})^2} \ldots [15]$$

From equation (15) it appears that the KE portion transferred from particle-1 to particle-2 is directly proportional to a term to the form of $(1+e)^2$ with the condition that μ_{21} is constant. As the value of e becomes smaller, the KE portion transferred from particle-1 to particle-2 also becomes smaller. This also applies to the opposite condition.

For find a sharper analysis for equation (15), it can be doing by analyzing the curve $\eta = f(e)$. The related curve shown in Figure 4, with a special condition where μ_{21} = 1. Appears that there are three special conditions found, namely: (a) The restitution coefficient is zero. This collision type includ in the inelastic category. Simulations show that only 25% of the initial KE can transfer. After the collision, the 2-particles move together; (b) The restitution coefficient meets the condition of 0 < e < 1. This collision type category is partially elastic. In this type, the condition applies 25% < portion of KE transferred between particles < 100%; and (c) The restitution coefficient is 1. This type of collision category is perfectly elastic. In this collision type, 100% of the KE of particle-1 transferred to particle-2.

Collisions with the condition η = 1, as explained in Figure 3 above, also appear implicitly in Figure 4. This means that to achieve the maximum value of KE transferred, the particle mass ratio and restitution coefficient, each of which must be one. This harmony with theory or phyisics concept that the restitution coefficient price must meet 0≤e≤1 where the maximum restitution coefficient value is only one.

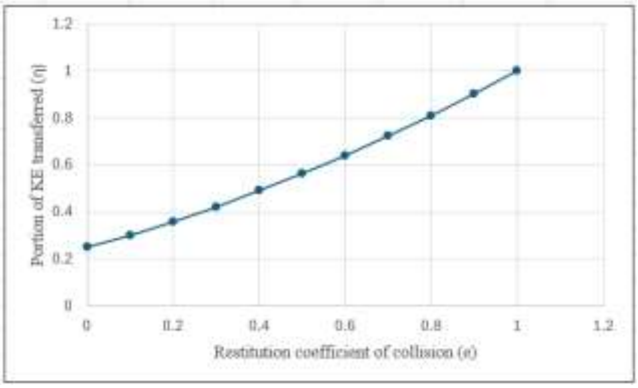

Fig. 4 Dependence of η on e for a 2-particles elastic collision with the special condition μ_{21}= 1.

When the restitution coefficient effect is analyzed further under several conditions of the particle mass ratio (μ_{21}) and in related with the 3 types of 2-particle collisions that exist (perfectly elastic, partially elastic, inelastic) it appears that the type of collision that produces the highest KE-transfer value is a perfectly elastic collision, followed by partially elastic collisions, and the lowest is inelastic collisions.

The final analysis of IM of the simple 2-particles elastic collision is about the particle-1 critical velocity which can produce conditions where KE transfer between particles is maximum. To analyze this, a framework approach is used as the basis for consideration, namely the particle-1 velocity before the collision, if it is increased smoothly (little by little), then in time a certain condition will be reached where the amount of KE-transfer between particles is maximum. The velocity of particle-1 at this condition called the critical velocity (v_{1c}). A similar framework approach can apply to the restitution coefficient change. This means that the particle-1 velocity before the collision (v_1) can reach a critical value when the coefficient of restitution is one. This reasoning can be expressed in a mathematical model form:

$$e = \frac{v_1}{v_{1c}} \ldots (16)$$

The result is that equation (15) above can restated as an IM mathematical model in the form:

$$\eta = \frac{EK_2{}'}{EK_1} = \frac{\left(1+\left({}^{v_1}/_{v_{1c}}\right)\right)^2 \mu_{21}}{(1+\mu_{21})^2} \ldots (17)$$

Appears from equation (17) that the particle-1 critical velocity before the collision for the maximum KE-transfer condition, is very related with the particle mass ratio μ_{21}. For the special condition of maximum KE transfer with using μ_{21} = 1, it can verified that the particle-1 velocity before the collision is the same as the particle-1 critical velocity before the collision or expressed as $v_1 = v_{1c}$. This proves that the qualitative reasoning for changes in particle-1 velocity reaching a critical value is correct and in accordance with the reasoning for changes in the restitution coefficient value from 0 to 1 where the KE-transfer has a maximum value.

CONCLUSIONS

IM of simple 2-particles elastic collision concerns the maximum KE transfer from particle-1 to particle-2. Theoretically, IMs influenced particle velocity ratio, particle mass ratio, and restitution coefficient of collision type. IM condition with critical conditions achieved if particle mass as the collision cause is the same as particle mass as the collision target, its collison condition must perfect elastic, and its particle-1 velocity, before and after collisions must the same. Theoretically, controlling act for collision-causing particles velocity of so achieved to its critical value only occurs when the masses of the two particles are the same and the collision of the two particles is perfectly elastic.

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