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Numerical Computations of Demographic Population Dynamics Models

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ABSTRACT: In this paper, the population dynamics models including the Environmental problems is of paramount importance which is the issue of controlling the population size of living organisms in complex ecological systems has become urgent. Mathematical models are needed that allow one to assess the possible consequences of human impact on nature and organize his activities in such a way as to prevent an "ecological catastrophe. Such problems can be solved if a mathematical model corresponding to the required situation is analyzed. In this work, we built, solved, and also investigated a model of unlimited population growth, a model of the predator-prey system according to the Lotka-Volterra problem.

KEYWORDS: Model of population dynamics, Lotka-Volterra problem, predator-prey system.

1. INTRODUCTION

An ecosystem is a complex collection of species interacting with each other, moving in space, and changing their numbers. Various abiotic and anthropogenic factors also affect all populations in an ecosystem. The population size depends on the weather, the chemical composition of the environment, and the degree of its pollution. Increasing human economic activity gradually leads to irreversible changes in many natural systems. Their restoration and rational use is one of the most important tasks today since the further prosperous existence and development of society is possible only in harmony with nature [1.2].

A special case of mathematical modeling is computer simulation, which allows us to analyze the characteristics of interactions between a population and environmental factors, consider various scenarios for changes in these factors, and make forecasts of population dynamics. Therefore, the issue of developing an adequate mathematical apparatus for studying population dynamics is relevant. Considering the importance of using mathematical modeling to study patterns of population dynamics [3,4]. Several works are devoted to the issues of modeling population dynamics in agricultural societies and exiting the Malthusian trap, however, in the proposed models, society is mainly considered in aggregate, without dividing into social strata. Meanwhile, it seems that for a more detailed analysis of both the Malthusian trap itself and the processes of exiting it, it is necessary to take into account the social structure of society. In several works basic mathematical models were proposed to describe dynamics in agricultural societies taking into account their social structure [5,6,7].

2. CONCEPTS OF CONSTRUCTION OF A POPULATION DYNAMICS MODEL

The recurrence formula for the terms of the Fibonacci series written down

$$\mathrm{T}_{n+2} = \mathrm{T}_{n+1} + \mathrm{T}_n \tag{1}$$

where T represents a member of the sequence, and the subscript n is its number in a series of numbers. Noticed that as the ordinal number of the terms of a series increases, the ratio of the subsequent term to the previous one approaches the number called the golden ratio which is equal to $(1+\sqrt{5})/2$. The Fibonacci series and its properties are also used in computational mathematics to create special counting algorithms. The second world-famous mathematical model, which is based on the problem of population dynamics, is the classical model of unlimited growth is a geometric progression in a discrete representation

$$\begin{cases} \mathrm{T}_{n+2} = \lambda \mathrm{T}_n \\ \frac{d\mathrm{X}}{dt} = \rho \mathrm{X} \end{cases} \tag{2}$$

Economic pessimism which follows from the forecasts of the proposed model, which is based on the analysis of empirical data, was contrasted by Malthus with the optimistic ideas of humanists that were fashionable. The result is a slowdown in the growth rate and it's reaching a stationary level. For the first time, the systemic factor limiting the growth of evolution was formulated by Verhulst in the logistic growth equation

$$\frac{d\mathrm{X}}{dt} = \rho \mathrm{X}(1 - \frac{\mathrm{X}}{\mathrm{K}}) \tag{3}$$

Equation (3) has two important properties, when X is a small, the number X increases exponentially, when K is a large, it approaches a certain limit. This quantity is called the population capacity. Thus, the capacity of an ecological niche is a systemic factor that determines the limitation of population growth in a given habitat. Equation (3) can also be rewritten as

$$\frac{dX}{dt} = \rho X - \beta X^2 \quad (4)$$

where β is the coefficient of intraspecific competition. Equation (3) can be solved analytically. The solution has the following form

$$X(t) = \frac{X_0 K e^n}{K - X_0 + X_0 e^n} \quad (5)$$

formula (5) describes the kinetic curve, that is, the dependence of population size on time. The Verhulst logistic model (3) turned out to be no less remarkable than the Fibonacci series. The study of this equation in the case of discrete changes in numbers in populations with non-overlapping generations showed a whole range of possible types of solutions, including oscillatory changes of different periods and outbreaks of numbers. Models that describe both intraspecific competition, which determines the upper limit of population size, and the lower critical population size, have two stable stationary solutions. The proposed nonlinear equation

$$\frac{dX}{dt} = \alpha \frac{\delta X^2}{\delta + \tau X} - \rho X - \beta X^2 \quad (6)$$

in equation (6), the first term on the right side describes the reproduction of a bisexual population, the rate of which is proportional to the square of the number (the probability of encountering individuals of different sexes) for low densities, and proportional to the number of females in the population for high population densities. The second term describes mortality proportional to abundance, and the third describes intraspecific competition. Volterra's mathematical theory of the struggle for existence proposed describing the interaction of species, similar to what is done in statistical physics and chemical kinetics, in the form of multiplicative terms in equations, that is, products of the numbers of interacting species. Then, in general, taking into account the self-limitation of numbers according to the logistic law, the system of differential equations describing the interaction of two species can be written in the form

$$\begin{cases} \frac{dX_1}{dt} = \alpha_1 X_1 - \rho_1 X_1 X_2 - \beta_1 {X_1}^2 \\ \frac{dX_2}{dt} = \alpha_2 X_2 - \rho_2 X_1 X_2 - \beta_2 {X_2}^2 \end{cases} \quad (7)$$

where the parameters $\alpha_{1,2}$ are constants of the species' growth rate, $\rho_{1,2}$ constants of self-limitation of numbers, $\beta_{1,2}$ constants of interaction of species. The competition model of type (7) has disadvantages, in particular, it follows that the coexistence of two species is possible only if their numbers are limited by different factors, but the model does not indicate how large the differences must be to ensure long-term coexistence.

3. NUMERICAL COMPUTATIONS OF POPULATION DYNAMICS MODEL

Pattern 1. Problem statement: build a model of unlimited growth of protozoa. A single-celled protozoa divides into two every 4 hours. Let's build a model of cell number growth after 4, 8, 12, 16,... hours. Factors leading to the death of protozoa are not taken into account, for creating a mathematical model, We use the formula for increasing time

$$T_{n+1} = T_n + A \quad (8)$$

where A is the time increase interval.

The calculation of the number of protozoa is calculated using the formula

$$T_{n+1} = T_n * B \quad (9)$$

where T_n is the number of protozoa in the first period, T_{n+1} is the number of protozoa at the $n + 1$ time point, B is the biotic potential of protozoa (it is equal to 2 for 4 hours). We will build a model of unlimited growth of protozoa, as shown in table 1 and figure 1 below.

Table 1. Calculations of the number of protozoa for B = 2.

n	0	1	2	3	4	5	6	7	8	9	10
Time per hour	0	4	8	12	16	20	24	28	32	36	40
number of protozoa	10	20	40	80	160	320	640	1280	2560	5120	10240

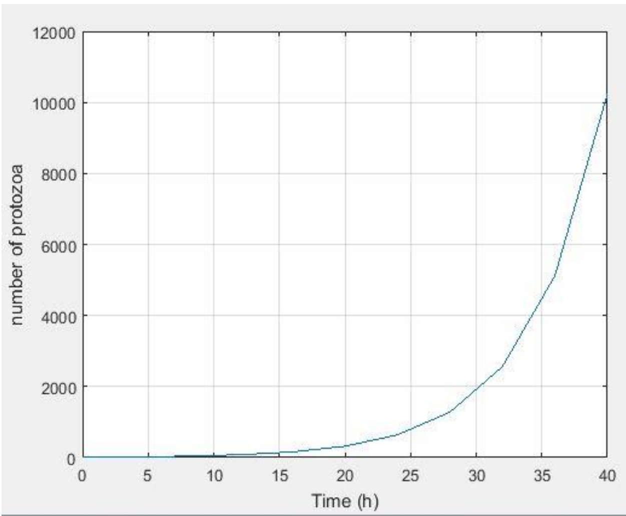

Fig. 1 Calculations of the number of protozoa for B = 2.

Pattern 2. The Lotka-Volterra equation as a mathematical model of the dynamics of the Predator-Prey system modeling population dynamics becomes more complex when trying to take into account the actual relationships between species. The model, now known as the Lotka-Volterra equation, considers the interaction of two populations - predator and prey. The population size of the prey will change over time according to the following equation

$$\frac{dX_1}{dt} = \alpha_1 X_1 - \rho_1 X_1 X_2$$

(10)

where X_1 is the size of the prey population, X_2 is the population size of the predator, α_1 is the coefficient fertility victim, and ρ_1 is the coefficient victim mortality. The growth of the predator population is described by the following equation

$$\frac{dX_2}{dt} = \rho_2 X_1 X_2 - \alpha_2 X_2$$

(11)

where X_1 is the size of the prey population, X_2 is the population size of the predator, ρ_2 is the coefficient of predator mortality, and α_2 is the coefficient of predator birth rate.

We simulate the dynamics of the two populations consisting of the generation number, the number of prey, and predator, the generation number naturally take 1, as well as the initial numbers of prey is equal to 100 individuals and predators is equal to 10 individuals, i.e.

$X^0{}_1 = 100$, $X^0{}_2 = 10$ which represent initial values, $\alpha_1 = 0.1, \rho_1 = 0.012, \rho_2 = 0.05$ and $\alpha_2 = 0.0008$.

We will establish the mathematical model of the dynamics of the Predator-Prey system modeling population dynamics at the initial values of 1000 and 100 individuals, respectively, that is, the populations are in equilibrium, the numerical computations as shown in table 2 and figure 2 below.

Table 2. Calculations of the mathematical model of Predator-Prey.

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
size of the prey	**20**	**40**	**60**	**80**	**100**	**120**	**140**	**150**
size of the predator	**600**	**1200**	**1500**	**1800**	**1900**	**2100**	**2200**	**2300**

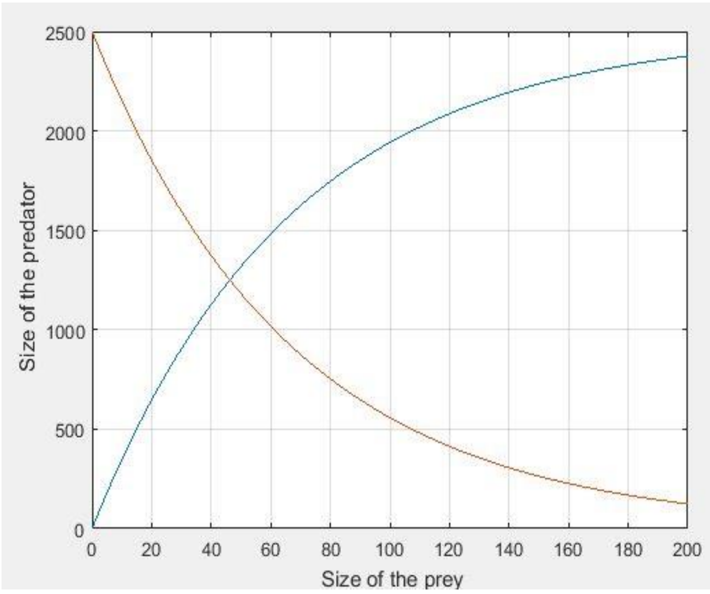

Fig. 2 Calculations of the mathematical model of Predator-Prey.

4. CONCLUSIONS

It is necessary to restore the population of animals that are on the verge of extinction, and there are times when it is needed to reduce the number of certain pests and maintain their population at a given level. In this case, it is necessary to take into account how certain changes in the number of one population are reflected in the number of individuals of other species in a given ecosystem. Such problems can be solved if a mathematical model corresponding to the required situation is analyzed. We built, solved, and also investigated: a model of unlimited population growth, a model of the predator-prey system according to the Lotka-Volterra equation, and a model of population dynamics. Thus, all the results obtained numerically are visual.

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