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An Optimized Approach to Solve Assignment Problems Using the Fuzzy Hungarian Method

Amit Kumar Nandan[1], J. P. Tripathi[2], V. K. Tiwari[3]

[1]Research Scholar, Department of Mathematics, B. R. A. Bihar University, Muzaffarpur, Bihar, India

[2]Department of Mathematics, R.N. College, Hajipur, B. R. A. Bihar University, Muzaffarpur, Bihar, India

[3]Department of Commerce, Government Degree College Sukrauli, Kushinagar, U.P., India

ABSTRACT: The assignment method assigns a particular (employee) to a particular (work) "Assuming an equal number of jobs and workers." In this paper, we are introducing a pentagonal ranking method by which we can convert the cost values (Fuzzy numbers) of the Fuzzy Assignment Problem (FAP) into crisp values. By the Hungarian method, we can optimize the cost of the FAP. A numerical value illustrates the method.

KEYWORDS: Fuzzy Set, Pentagonal Fuzzy Number, Fuzzy Assignment Problem (FAP), Pentagonal Ranking Function, Hungarian Method.

I. INTRODUCTION

The FAP stands as a powerful and effective solution for tackling real-world challenges on a global scale. In 1965 Zadeh was the first who introduce "The concept of fuzzy set theory." The FST can be used in operation research, control theory, neural networks, finance, etc.

The assumptions made in the assignment problem are as follows:

- Each person is assigned to only one job.

Lin and Wen [1] introduced an effective approach for solving linear fractional programming by employing the labeling method. The assignment problem's cost matrix elements are depicted as subnormal fuzzy intervals utilizing increasing linear membership functions. In contrast, the total cost's membership function is a fuzzy interval with a decreasing function.

The normalization process is a major drawback when it comes to analyzing data. Due to the transformation of objective measurements into subjective valuations, the loss of valuable data can occur. This procedure is very accurate and can be used in a variety of ways. However, it should be noted that reducing the amount of data in the original dataset should not be considered. The concepts of fuzzy sets, fuzzy nos, and fuzzy linear programming were addressed by S. V. Gomathi et al. [3]. Thakre et al [4] have offered valuable insight into several methods for achieving the lowest rate. K. Kalaiarasi, S. Sindhu, and M. Arunadevi [5] proposed a method using the robust ranking method and Hungarian algorithm to solve the triangular fuzzy assignment problem. Dhanasekar et al. [6] incorporated the Haar ranking technique into the Hungarian algorithm to effectively address the FAP. Pramanik and Biswas [7] conducted a study on a multi-objective Ass. Prob. Military domain, considering the quality, time, and cost represented as Trapezoidal F. Nos. They assigned weights to the objectives based on their attributes and used Yagar's (1981) ranking method to merge the objectives into a single problem. In [8], the author discusses various strategies for solving an Unbalanced A.P. The proposed method ensures the optimal acquisition of an IBFS using the Row Penalty or Column Penalty Allocation Method. Symmetric interval-valued fuzzy nos. are a notion that is introduced by A. B. et al. [9]. The fuzzy costs of FAP are represented by two types of fuzzy nos.: symmetric trapezoidal interval-valued and symmetric triangle interval-valued. The positional technique is then used to transform these values into crisp values. The fuzzy technique method uses the idea of a linguistic variable for analysis. A novel fuzzy goal programming technique based on priorities and designed for generalized trapezoidal FNs was presented by P. and B. [10]. In multi-objective APs, they used it on the assumption that time and cost are generalized trapezoidal FNs. Ghadle and Pathade [11] have introduced an innovative approach, known as One's BCM method, to effectively address the transshipment problem. Solving assignment problems by the Hungarian method was developed by H. W. Kuhn [12] in 1965. A. K. Nandan, J. P. Tripathi et al [13] find the optimal solution of Assignment Problems using fuzzy ranking technique.

II. DEFINITIONS

Fuzzy Set: Assuming a Universal set Y, a set Ã is considered a fuzzy set if it is mapped from Y to the closed interval [0,1] by a membership function. Its definition is the collection of pairings provided by

$$\tilde{A} = \{(y, \mu\tilde{A}(y)); y \in Y\}$$

Where μÃ(y) is the membership value of y ∈ Y of the fuzzy set Ã and μÃ: Y → [0,1] is the degree of membership function of the fuzzy set Ã.

The membership value is given by

$$\mu_{\tilde{A}}(y)=\begin{cases} 1, \text{if } Y \text{ is totally in } \tilde{A} \\ 0, \text{ if } y \notin \tilde{A} \\ (0,1), \text{if } Y \text{ is partially in } \tilde{A} \end{cases}$$

III.KINDS OF FUZZY NUMBERS

Fuzzy numbers can be found in the following forms: Triangular, Trapezoidal, Pentagonal, Hexagonal, Octagonal, etc.

Here, we discuss pentagonal fuzzy number.

IV. PENTAGONAL FUZZY NUMBER

The definition of a linear pentagonal fuzzy number is $\tilde{A}_{LS}$ = (a, b, c, d, e; s) with the following membership function which is as follows.

$$\mu_{\tilde{A}_{LS}}(y) = \begin{cases} s\frac{(y-a)}{(b-a)} & \text{if } a \le y \le b \\ 1-(1-s)\frac{(y-b)}{(c-b)} & \text{if } b \le y \le c \\ 1 & \text{if } y = c \\ 1-(1-s)\frac{(d-y)}{(d-c)} & \text{if } c \le y \le d \\ s\frac{(e-y)}{(e-d)} & \text{if } d \le y \le e \\ 0 & \text{if } y > e \end{cases}$$

V. RANKING OF PENTAGONAL FUZZY NUMBER

If $\tilde{A}_p$ = (a, b, c, d, e; s) is a fuzzy number, then the ranking technique of the pentagonal fuzzy number may be found by

$$R(\tilde{A}_p) = \frac{1}{4}\,(2a + 3b + 2c + 3d + 2e)$$

VI. ARITHMETIC OPERATIONS

Let us assume two Pentagonal Fuzzy Nos.

$\tilde{A}1_{LS} = (a_1, b_1, c_1, d_1, e_1; s_1)$ and
$\tilde{A}2_{LS} = (a_2, b_2, c_2, d_2, e_2; s_2)$

❖ **Addition**:

$$\tilde{A}1_{LS} + \tilde{A}2_{LS} = (a_1, b_1, c_1, d_1, e_1; s_1) + (a_2, b_2, c_2, d_2, e_2; s_2)$$
$$= (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2; S)$$

Where $S = \min\{s_1, s_2\}$

❖ **Subtraction**:

$$\tilde{A}1_{LS} - \tilde{A}2_{LS} = (a_1, b_1, c_1, d_1, e_1; s_1) - (a_2, b_2, c_2, d_2, e_2; s_2)$$
$$= (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2, e_1 - e_2; \boldsymbol{S})$$

Where $S = \min\{s_1, s_2\}$

VII. FUZZY ASSIGNMENT PROBLEM CONCLUSIONS

It may be given by the (n x n) fuzzy cost matrix
$C_{ij} = [c_{ij}]_{nxn}$.
The primary goal is to find the optimal solution by assigning the i^{th} resource to the j^{th} job by minimizing the total cost and time.

VIII. MATHEMATICAL FORMULATION OF FUZZY ASSIGNMENT PROBLEM

It is as follows

$$\text{minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} Cijyij$$

subject to

$$y_{ij} = \begin{cases} 1, & \text{if ith resource is assigned to jth job} \\ 0, & \text{otherwise} \end{cases}$$

IX. PROPOSED ALGORITHM

Step 1: First, convert the cost values represented as pentagonal fuzzy numbers of the FAP into crisp values by using a Pentagonal Ranking Technique**.**

Step 2: Determine whether the condition of FAP is balanced or not.

i. Proceed to step 4, if the number of rows and columns is equal. It is considered balanced.

ii. Proceed to step 3, if the number of rows and columns is not equal. This indicates that the arrangement is not balanced.

Step 3: for the balanced FAP we introduce the dummy rows or dummy columns with zero cost value.

Step 4: Solve the fuzzy cost matrix by Hungarian method to get the optimal fuzzy assignment.

X. EXAMPLES

A FAP in which the rows represent 4 resources R_1, R_2, R_3, R_4 and the columns represent 4 jobs J_1, J_2, J_3, J_4. Then the cost matrix C_{ij} with the elements Pentagonal Fuzzy Numbers is as follows.

To find the optimum cost of the cost matrix.

Cost Matrix-1

$(1,3,5,6,7)$	$(5,6,7,10,11)$	$(2,3,5,9,11)$	$(16,18,20,21,23)$
$(5,6,7,10,11)$	$(12,15,17,18,19)$	$(14,16,18,20,22)$	$(12,15,17,18,19)$
$(3,5,7,11,12)$	$(1,3,5,6,7)$	$(17,19,23,24,25)$	$(5,6,7,10,11)$
$(14,16,18,20,22)$	$(10,11,12,14,15)$	$(1,5,9,11,13)$	$(3,5,7,11,12)$

Convert the fuzzy numbers of the cost matrix into crisp values by using a ranking technique

$$\frac{1}{4}\,(2a + 3b + 2c + 3d + 2e)$$

We get,

Cost Matrix -2

13.25	**23.5**	**18**	**58.75**
23.5	**48.75**	**54**	**48.75**
23	**13.25**	**64.75**	**23.5**
54	**37.25**	**23.5**	**23**

After column-wise subtraction

We get,

After row-wise subtraction

We get,

Cost Matrix -3

0	**10.25**	**4.75**	**45.5**
0	**25.25**	**30.5**	**25.25**
9.75	**0**	**51.5**	**10.25**
31	**14.25**	**0.5**	**0**

Cost Matrix -4

0	**10.25**	**4.7**	**45.5**
0	**25.25**	**30**	**25.25**
9.75	**0**	**51**	**10.25**
31	**14.25**	**0**	**0**

Now, the Fuzzy cost matrix is

Cost Matrix -5

0	**5.55**	**0**	**40.8**
0	**20.55**	**25.3**	**20.55**
14.75	**0**	**51**	**10.25**
35.7	**14.25**	**0**	**0**

Since all the resources will not get the job, so we draw a straight line on Zeros.

i.e. Column 1, row 3, and row 4 (We get the intersection point is c_{13} and c_{14})

By deleting these cells, we need to find the minimum cost from undeleted cells.

Therefore, the minimum value is 4.7

Add the minimum value (4.7) to the intersection point only and subtract the minimum value (4.7) from undeleted cells.

We get,

Cost Matrix -6

0	5.55	0	40.8
0	20.55	25.3	20.55
14.75	0	51	10.25
35.7	14.25	0	0

By the Hungarian method, the optimal allocations are

Cost Matrix -7

0	5.55	(0)	40.8
(0)	20.55	25.3	20.55
14.75	(0)	51	10.25
35.7	14.25	0	(0)

Therefore, the assignment is

Table-1

Resources	Jobs
R_1	J_3
R_2	J_1
R_3	J_2
R_4	J_4

Hence, the assignment cost = 18 + 23.5 + 13.25 + 23
= 77.75

CONFLICT OF INTEREST: No conflict of interest.

CONCLUSION

We considered the fuzzy assignment problem's fuzzy numbers as pentagonal. By using pentagonal ranking, we converted the pentagonal fuzzy numbers into crisp values. With the help of the Hungarian method, we optimized the fuzzy assignment problem to get the final optimal assignment.

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