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Physics Applied to the Sailboat Optimum Upwind Path Algorithm

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ABSTRACT: Autonomous vehicles are predicted to be an enormous disrupter to our economy, creating a massive and unpredictable impact, much like the smartphone on telecommunications. Marine transportation is at the leading edge, with automated boats crossing the oceans. Sailboats and sail-assisted boats, in particular, are not just efficient and environmentally friendly, but also allow automated boats to be at sea for extended periods. This is excellent for applications such as oil and gas pipeline monitoring, data collection, and ocean mapping. Navigation is critical, however, one issue unique to sailboats is that it is impossible to sail directly into the wind. In order to navigate upwind the boat needs to follow a zig-zag route towards the destination. Details such as the angles, lengths, and number of zig-zags are critical to maximizing forward progress to the destination. This research includes an approach to determine the optimal zig-zag route for efficiency, which could be used as part of route planning before a journey begins, or as part of an autonomous sailboat system.

KEYWORDS: Autonomous, Sailboat, Sailing, Upwind, Tacking, Marine Navigation, Efficient, Python, No-sail Zone, Heading, Polar Plot

INTRODUCTION

Sailing directly into the wind is impossible for a sailboat because the sail cannot collect air to generate force, creating a zone directly in front of the boat called the no-sail zone [1]. The no-sail zone is a result of how sails generate force. A common assumption is that sails simply collect the energy from the wind pushing them and move in that direction, which is true only when the wind is coming from directly behind the boat. In most other cases, the sail acts much more like an airplane wing, and if you look closely at a sail, you will notice once it is full of air it becomes shaped like a wing [2]. The sailor can control the shape of the wing by controlling the tension on the mainsail. When shaped like a wing, both sides of the sail have air flowing over them. One side has positive pressure and the other has negative pressure generating lift, which works together to both push and pull the boat [3]. Sailboats also need a board or keel that goes down into the water typically in the center of the boat to help ensure that a wind force acting on the side of a boat gets translated into a force pushing the boat forward [4]. When the wind is coming directly towards the boat, the pressure on both sides of the sail is roughly equal, and the sail cannot maintain its wing shape, so it flaps around more like a flag, referred to as luffing. This is called the no-sail zone, and the boat needs to change its angle until there is enough pressure difference for the sail to maintain its wing shape. The zone close to either side of the no-sail zone is a point of sail referred to as close-hauled. To change direction, and come about during the tacking, the rudder needs to be controlled. The rudder is an underwater fin that deflects water and is controlled by the sailor to steer the boat and set the desired heading [3].

The no-sail zone is different for every boat. The sailboat transform is a chart on a polar plot that shows how fast a boat will be moving for a given wind speed and wind direction, one example is shown in Figure 2. In a polar plot, the axis is circular, beginning with 0° at the top, going anti-clockwise to 360°. The no-sail zone is easily visible since it shows a boat speed of near zero or is not shown at all when the wind direction is directly ahead near 0°. Sailboats are designed to be symmetrical, which leads to the sailboat transformation being symmetrical and typically only being shown for one half, port or starboard [5]. Once the optimum heading is determined for one side, it would be the same for the other.

This research aims to determine the most appropriate strategy for a sailboat to travel upwind and use that information to include in a navigation module for an autonomous sailboat. This information is valuable even for someone manually planning the route of a sailboat trip but is vital for an autonomous system [6-7]. The no-sail zone could be anything from 20-50° on either side of the center line. The width of the no-sail zone depends upon many factors relating to the boat's design [4,8]. To head upwind, a boat needs to zig-zag, referred to as tacking, so that the sail is always at an angle to the wind greater than the no-sail zone [2], as shown in Figure 1. The direction the boat is pointing is referred to as the heading, so the angle of the heading must be greater than the no-sail zone [9]. The heading can be used to calculate the forward progress towards the destination, as shown in Figure

3. The common assumption is that the most direct path would be sailing with an angle as close as possible to the no-sail zone with a narrow channel since that is the closest to a direct route [3]. The channel is the width of the path the boat makes from side to side as the boat zigzags upwind. The decisions a sailor or autonomous system needs to make are what angle or heading should be selected for the tack, and how long the tack is maintained [10]. The length of the tack determines the width of the channel made by tacking and the number of times the boat needs to come out and head in the opposite direction. This research aims to determine the optimum values for these variables.

An important part of determining the fastest path will be manipulating velocity vectors. The wind direction experienced by the boat, that is, the wind relative to the boat's direction dictates the boat's forward speed [11], which is a series of lines plotted on a polar plot [12]. The boat's velocity can be considered as made up of a vertical component heading directly towards the destination, and a horizontal component [13]. The closer the heading is towards the destination, the more of the boat's velocity is going towards the destination as shown in equation 2.

This research determined the optimum strategies for sailing upwind which will be added to a navigation module to best set a path to the destination. The navigation module is part of an autonomous sailboat, first developed when I was learning to sail. I wanted something to help me know the best angle to set the sail based on the wind speed and direction, and I designed and created a control system with integrated wind speed, direction, GPS, IMU, and Dead Reckoning system to control an autonomous sailboat [14]. I had made my earlier research reports available to help others, in the same way that I had been helped by others. I would like to publish this work to continue that path of helping others.

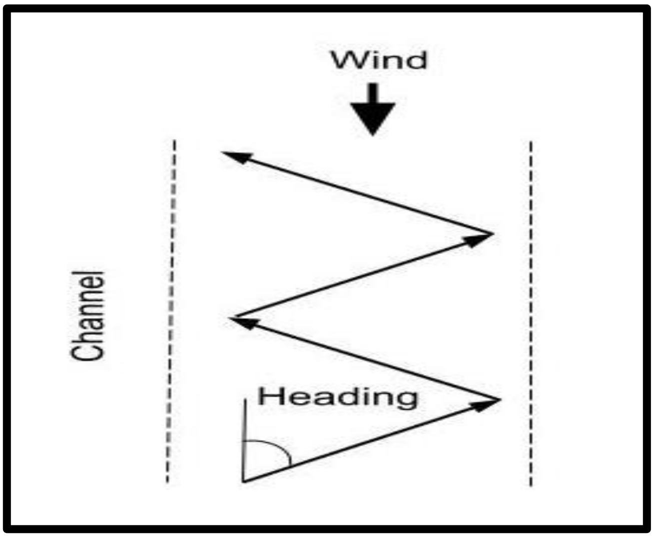

Figure 1. Boat tacking upwind: A sailboat cannot sail directly upwind and must sail at an angle zig-zagging upwind. The selected angle (heading) must be greater than the no-sail zone. The length of the tack will determine how many tacks are needed to reach the destination, and how wide the channel is that the sailboat is creating.

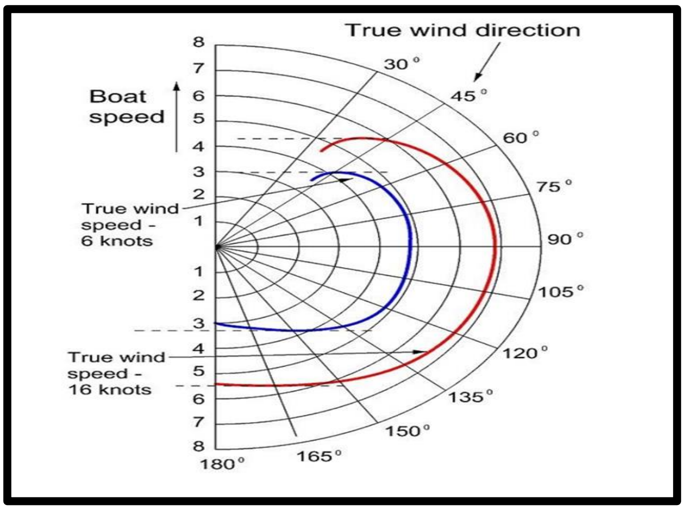

Figure 2. Sailboat Transform Polar Plot: The axis is circular, beginning with 0° at the top, going anti-clockwise to 360° in 15° increments. This sample includes two plots; a 16-knot wind speed is in red, and a 6-knot wind speed is shown in blue. With a true wind direction of 45°, looking at the intersection with the red line which indicates a wind speed of 16 knots,

this point intersects with a boat speed of 6 knots. When the plot is symmetrical, only half of the plot is shown from 0-180°. Image taken from [15].

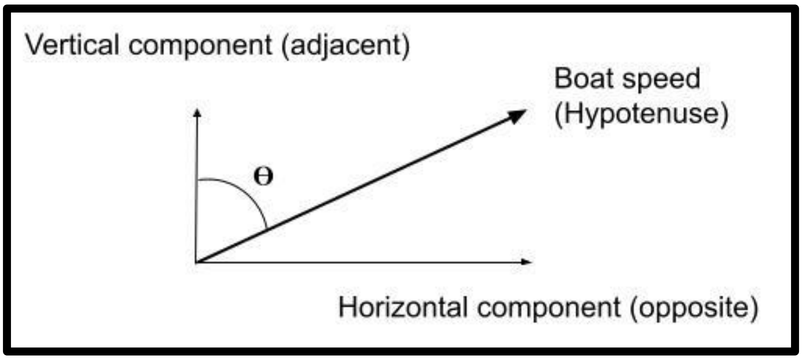

Figure 3. Components of the boat speed vector: Boat speed when tacking can be considered to be made up of two components, a vertical component which is the speed towards the destination, and a horizontal component which is perpendicular.

MATERIALS AND METHODS

Analyzing a boat transformation, such as that shown in Figure 2 or Figure 4, as the boat heads closer to 0°, the boat speed can fall dramatically. The boat speed may also increase as it gets closer to 90°. However, as the heading angle increases the further the boat is moving away from the destination the longer the distance traveled will be, until ultimately at 90° or above the boat would never arrive at the destination [16].

The relationship between distance traveled in a given tack can be seen in Figure 5, and given by

Distance traveled = vertical distance/cos (heading) --1

A derivation of this for the forward speed, and the boat speed and heading is given by

Forward speed = boat speed * cos (heading) --2

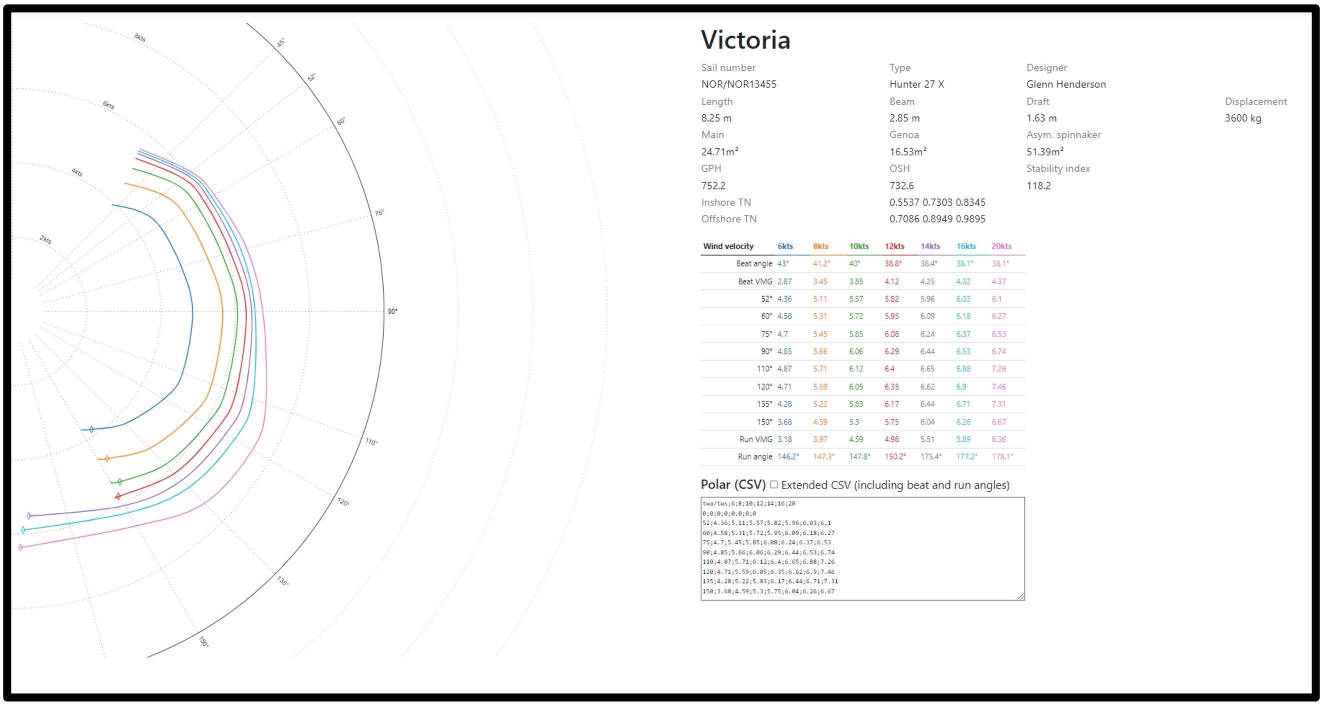

Figure 4. Hunter 27 Polar Plot: The Hunter 27 monohull sailboat transforms. The boat speed is shown for a range of wind speed from 6 knots to 20 knots. The plot is symmetrical, only half of the plot is shown from 0-180°. Image taken from [17].

Sailboats are designed to be symmetrical, which leads to the sailboat transformation being symmetrical and typically only being shown for one half, port or starboard [12]. Once the optimum heading is determined for one side, it would be the same for the other, which dictates that the optimum path is symmetrical. The common assumption is that a narrow channel is the most direct path, however, the narrower the channel the more tacks are needed, and a tack reduces velocity. Tacking involves going from one point of sail on one side (e.g. close-reach) to the same point of sail on the other side, going through the no-sail zone. The boat will always reduce velocity 'coming about' [18].

The first analysis is to determine which path is shorter, a more direct path with a narrower channel, or a wider channel with fewer tacks. A Python program vector.py shown in Figure 6 was developed which calculated and charted three different paths to a destination at a given heading, and displayed the total distance traveled together with a chart. These were developed using numpy and matplotlib libraries and equation 1. A destination directly in front (0°) at a distance of 100m was selected. Three different samples were used, with 1, 3, and 5 tacks. Headings 30, 45, and 70 degrees were used, and the total distance travelled was recorded for each heading. The sailboat transforms for a Hunter 27 shown in Figure 4 for a given speed of 10 knots was used to determine the boat speed [17].

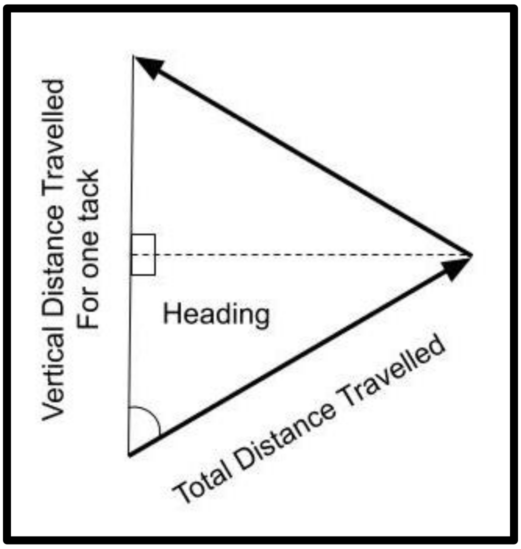

Figure 5. The vertical distance traveled for one tack is simply the distance to the destination divided by the number of tacks.

```
vector.py > ...
import numpy as np
import math as math
import matplotlib.pyplot as plt

# goal is 100m due north, directly in front
end = 100
# default heading 45 degrees
heading = math.radians(70)

y1 = end/2
x1 = y1*math.tan(heading)
v1a= np.array([x1, y1])
v1b= np.array([-x1, y1])
length1 = 2*np.linalg.norm(v1a)
print ("component1",v1a+v1b)
print ("length1", length1)
ax = plt.gca()
ax.quiver(0,0,x1,y1,color='b', angles='xy', scale_units='xy',scale=1)
ax.quiver(x1, y1, -x1, y1,color='b', angles='xy', scale_units='xy',scale=1)
ax.set_xlim([0, 150])
ax.set_ylim([0, 100])
```

Figure 6. Vector.py fragment for a single tack route. This example was for a 70° heading.

The second analysis is to determine the best heading. A python program polar.py highlighted in Figure 7, was developed which has a sailboat transform array for a Hunter 27ft monohull sailboat obtained from the ORC Sailboat database [17], as shown in Figure 4. The boat speed is displayed on a polar plot for 0-90° heading for a 10-knot wind. The forward speed was calculated for each boat speed using Equation 2. These results are then tabulated and shown graphically [19] to allow comparison of the forward speed to the boat speed.

```
polar.py > ...
import numpy as np
import math as math
import matplotlib.pyplot as plt


r = np.arange(0, 8, 0.01)
theta1 = [0.70, 0.91, 1.05, 1.31, 1.57]
theta = 2 * np.pi * r
r1 = [5.03, 5.57, 5.72, 5.85, 6.06]
r2 = [r1[0]*math.cos(theta1[0]), r1[1]*math.cos(theta1[1]), r1[2]*math.cos(theta1[2]),
print ("Apparent forward speed", r2)

fig, ax = plt.subplots(subplot_kw={'projection': 'polar'})
#ax.plot(theta, r)
ax.plot(theta1, r1)
ax.plot(theta1, r2)

ax.set_rmax(8)
#ax.set_rticks([0.5, 1, 1.5, 2])  # Less radial ticks
ax.set_rlabel_position(-22.5)  # Move radial labels away from plotted line
# extra
ax.set_theta_offset(np.pi/2)
ax.set_theta_direction(-1)
```

Figure 7. Polar.py fragment for a single tack route. This example was for a 70° heading.

RESULTS

After performing my tests by executing the two programs - vector.py and polar.py - I was able to gather and tabulate the results here, along with capturing the charts to help visualize the data. The first test was for distance traveled, and Table 1 shows the total distance traveled for a range of headings and a range of tacks. Figure 8-10 show the routes plotted for each of these combinations. For any given angle, regardless of the number of tacks, the distance covered is the same. The closer the heading is to the no-sail zone (the smaller the angle), the shorter the total distance traveled.

Table 1. Total distance traveled for different numbers of tacks and headings.

	Heading (degrees)		
Number of tacks	30°	45°	70°
1	115.47m	141.42m	292.38m
3	115.47m	141.42m	292.38m
5	115.47m	141.42m	292.38m

Table 2. Boat speed and forward speed for a range of headings with a 10-knot wind speed.

Heading (degrees)	Boat Speed (kts)	Forward speed (kts)
40°	5.03	3.85
52°	5.57	3.42

Heading (degrees)	Boat Speed (kts)	Forward speed (kts)
60°	5.72	2.85
75°	5.85	1.51
90°	6.06	0

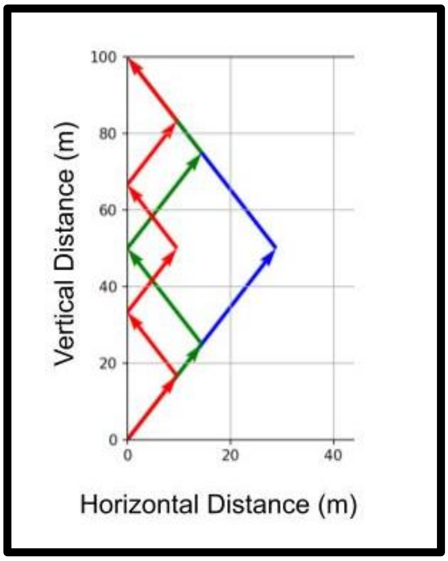

Figure 8. Vector.py output for 30° heading. The blue line shows a single tack, the green line shows three tacks, and the red line shows five.

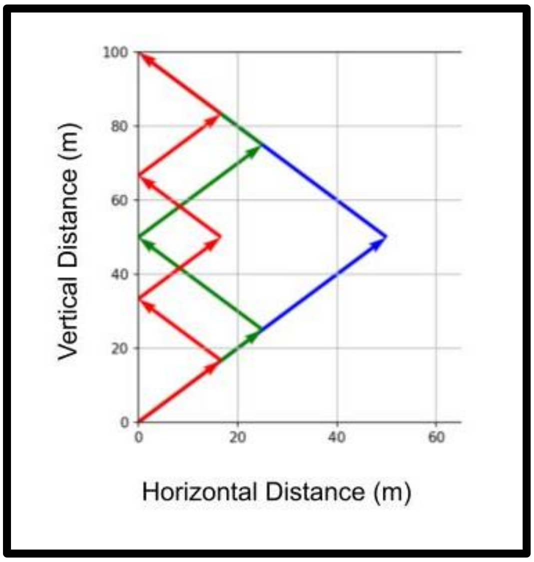

Figure 9. Vector.py output for 45° heading. The blue line shows a single tack, the green line shows three tacks, and the red line shows five.

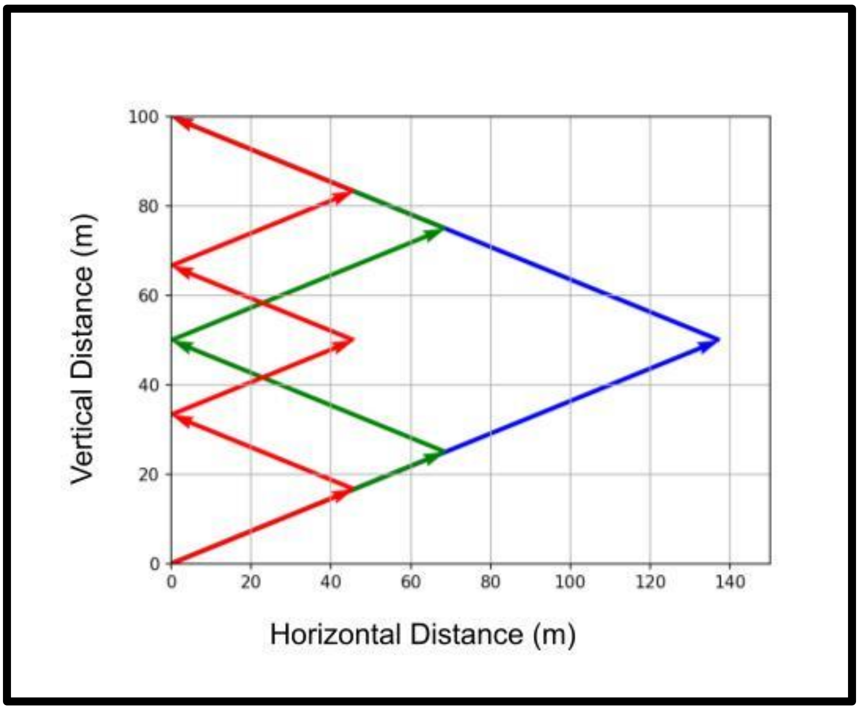

Figure 10. Vector.py output for 70° heading. The blue line shows a single tack, the green line shows three tacks, and the red line shows five.

The second test was to determine a boat's forward speed for a range of headings. Table 2 shows the boat's forward speed obtained for a given wind speed of 10 knots. These are plotted on a polar plot shown in Figure 11. The highest forward speed is achieved with the lowest angle, the one closest to the no-wind zone, even though this is not the highest boat speed.

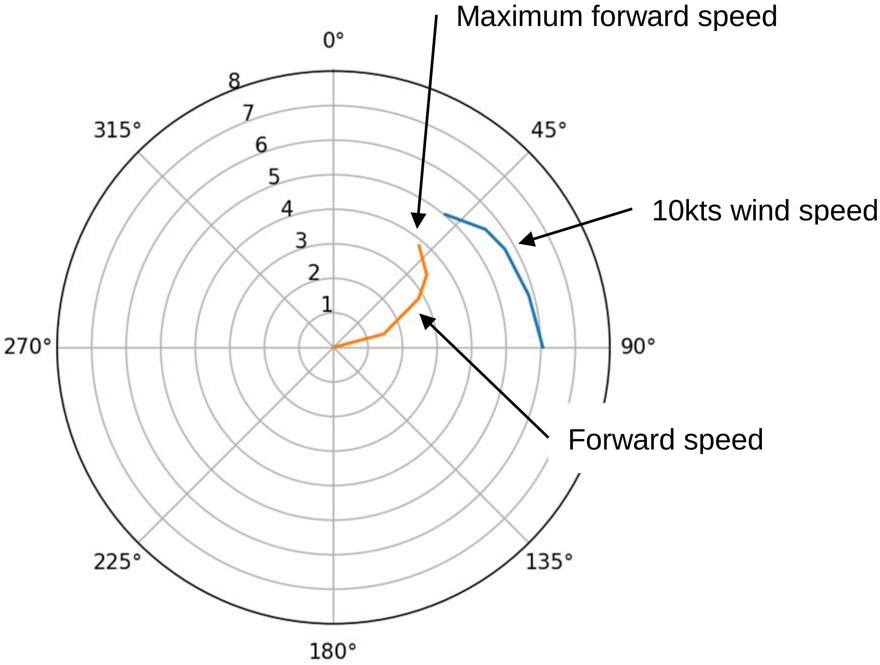

Figure 11. Polar plot. The blue line shows the boat's speed compared to the heading for a fixed 10-knot wind speed. The orange line overlays the boat's forward speed for the same boat speed, wind speed, and heading combination.

DISCUSSION

Recognizing that the zigzag should be symmetrical due to the symmetrical nature of the polar plot was an important initial finding, which explains why most text shows a symmetrical zigzag without explanation. The clear finding that the optimum zig-zag pattern is the minimal number of tacks was surprising because the novice sailors and most sailors attempt to keep a tight path to the upwind destination, which is incorrect. Coming about, which is the turn in a tack, requires going through the no-sail zone, which is certain to reduce speed, the only question is by how much. Since the distance traveled test proved that for any given heading the total distance was the same for the increasing number of tacks, the path with the fewest tacks is the best. Minimizing the number of tacks would show that the optimal length of tack would be coming about half the distance to the destination. If based on the heading and distance to the destination this is not possible, the optimum length of tack would be the closest to the edge of the channel while allowing a whole number of tacks to the destination to maintain symmetry.

The next important finding is the optimum heading. The nearest heading to the no-sail zone is the shortest path. Some sailors recognize that the boat is faster at a larger heading and may believe that the additional speed will overcome the more direct path. The Hunter 27 polar plot has a typical shape for most sailboats, and Table 2 shows that changing the heading from 40° to 60° does indeed increase the boat speed by 14% (5.72/5.03) but reduces the forward speed dramatically by 26% (2.85/3.85). As can be seen from Figure 11, the boat speed would have to increase unrealistically to offset the extra distance the boat must travel, therefore the optimum heading is indeed the lowest. The resulting code fragments from vector.py and polar.py could be incorporated into the navigation logic of an autonomous sailboat to optimize upwind sailing [1].

CONCLUSION

This project was to determine the factors needed for an algorithm to determine the optimum path for a sailboat to head upwind since a sailboat cannot head directly upwind toward the destination and must zig-zag. The project included two experiments, both built around Python programs to create reusable code and display results. The first experiment was to determine the optimum number of tacks and therefore width of the channel. This was determined to be the minimum number of tacks even if that resulted in the widest channel. The second experiment was to determine the optimum heading, which was determined to be the lowest, even if that is not the heading with the fastest boat speed. The final important note is that the zig-zag pattern should be symmetrical to ensure the optimum heading is in use at all times.

Given the destination location, wind speed, wind direction, channel limit, and the sailboat transform an algorithm to calculate the optimum path upwind was determined. Calculations were performed using Python ready to be integrated into a navigation system. The next step is to build a Python sailboat transform, ideally self-learning.

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