

The IPO Underpricing Impact Analysis of Split Share Structure Reform in China

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Abstract When all the stocks are circulating in a hot market, Ljungqvist, Nanda, Singh(2003)(referred to as LNS model) first researched the IPO underpricing based on investor sentiment and proved that the optimal decision of underwriter is to offer stocks by many times. IPO underpricing is to compensate for the risk that the hot market will end too early. Combining LNS model with the split share structure in China, this paper researches the impact of split share structure on IPO underpricing, and poses a systematic theoretical model and proves the rationality of theoretical model through analysis of empirical. First, we analyze the case that the issuer sells IPO stock directly (direct selling) based on split share structure. The issuer is best to offer shares with many times to get a higher profit. Second, we also analyze the case that the issue chooses underwriter to sell IPO stock (underwriting) when the stock market exists split share structure. This case is divided into two cases: the underwriter has no non-tradable shares or has some non-tradable shares. The underwriter is also best to offer shares with many times. Compute the models of IPO underpricing of the two cases with maximum profit of the issuer and underwriter's participation constraint. Through the models, we can see that the reform of split share structure does reduce the size of IPO underpricing with some requirements on the price and quantity of non-tradable shares. This also promotes Chinese stock market to further developing and mature.

Keywords Split share structure; IPO underpricing; Hot market; Investor sentiment
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1 Introduction

IPO underpricing (initial public offering) is a phenomenon that the initial issue price of newly issued shares in the primary market is lower than its listing price (the closing price on the first day) in the secondary market. Also, some researchers said it is IPO overpricing, which is a phenomenon that the listing price of newly issued shares in the secondary market is higher than its initial issue price in the primary market. Some researchers do the study to the ratio in a few areas, which are shown in table 1.1 below.

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Table 1.1. The Rate of IPO Underpricing in All Kinds of Countries or Regions

Country	Researchers	Sample	Duration	Rates of IPO Underpricing
Canada	Jog, Riding	500	1971-1999	6.3%
Australia	Lee, Taylor	381	1976-1995	12.1%
England	Dimson, Levis, et al	3042	1959-2000	17.5%
America	Ibbotson, Sindelar, et al	14760	1960-2000	18.4%
Philippines	Sullivan, Unite	104	1987-1997	22.7%
India	Krishnamurti, Kumar	98	1992-1993	23.9%
Singapore	Lee, Taylor, Walter	128	1973-1992	31.4%
Korea	Dhatt, Kim, Lim	477	1980-1996	74.3%
Brazil	Aggarwal, Leal, et al	62	1979-1990	78.5%
Malaysia	Lsa, Yong	401	1980-1998	104.1%
China	Li Xiang, et al	743	1997-2004	132.33%
China	Zhunan, Zhuoxian	160	2001-2003	125.18%
Thailand	Wethyavivorn, et al	292	1987-1997	46.7%

In Table 1.1, the external data comes from Jay, R. Ritter "Investment Banking and Securities Issuance" in *Handbook of Economics of Finance* edited by George Constantinides; Milton Harris and Rene Stulz, forthcoming. The domestic data comes from the relevant references. The special factors of systems in China have a great impact on IPO underpricing, and it leads to a higher IPO underpricing in China than other countries. The factors of systems include the split share structure, government regulation and so on. In this paper, we only study the influence of the split share structure. The split share structure means that the stock in the listing firms is divided into two kinds (tradable shares and non-tradable shares). The tradable shares refer to the stock in the listing firms that is offered publicly and the shareholder can transact in the bond exchange. On the contrary, the non-tradable shares which can not be transacted in the secondary market refer to the stock that the publisher sell to the investors through private transfer agreement signed, and it includes the country share, legal person's, the inside staff's and so on. Most shares of firms are non-tradable and these non-tradable shares account for two thirds of all the stocks. However, the shares of public offering are only a few parts. The non-tradable shares can affect the value of the shareholders because the cost of the two kinds of stocks is different. The split share structure only exists in China. The split share structure results in that the price of the non-tradable shares is far below the price of the tradable shares in the secondary market, because which the investors are indifferent to the tradable shares and the investors to buy public offering shares will suffer a loss. The structure has a great impact on the issued firms which always achieve the asset allocation through annexing the transaction of stock right in the secondary market and is harmful to the stock market in China which wants to be maturer. To solve the factor, the split share structure reform was carried out in Chinese market in 2005. The government transform the transaction from the non-tradable shares to the tradable shares. Then, the value of tradable shares is protected and the evidence of progress in the stock market of China is also evident.

1.2 The Current Research Situations of Domestic and External

IPO underpricing is early found by Ibbotson [1]. He found that the average price of the new stocks after a month in the secondary market is higher about 12.87% than in the

primary market through researching the stock price of initial public offering from 1960 to 1969. In addition, he did the careful research on both the price and put forward 6 reasons to explain this phenomenon. Later, the phenomenon was called "the riddle of new-stocks". In recent decades, there are many foreign researchers doing theoretical analysis to this phenomenon. Of these, the system research is various models based on the information asymmetry between issuers, underwriters and investors. Generally, it can be divided into the following categories:

(1) The information asymmetry between issuers and underwriters "Investment banking monopoly information hypothesis" is put forward by Baron [3]. He think that because of the information asymmetry between issuers and underwriters, the issuers are willing to underprice issue stock to the underwriters. The investment banks (underwriters) can exposure more directly to the stock market and know more about the demand information in the stock market than the issuers. In the course of pricing, the issuers can not see if the underwriters try their best to promote the stock to the secondary market, so the risk averse (the underwriters) hope the issuers can issue the stock in more underpricing to ensure the stocks in the secondary market are issued smoothly and they can capture more profits.

(2) The information asymmetry between investors The most classical study in this part is "Winners' Curse" which is posed by Rock [4]. He thinks that the information which is mastered by the investors is asymmetry. Some investors who are called "the investors in the know" or "the investors rich in information", know about the prospect of the company and the information of the quality overall. Other investors are not comprehensive and called "the unknown investors" or "the investors short of information". The first investors only buy the cheap stocks, but the second investors may buy the expensive stocks because of the incomprehensive information. It will bring in a risk or loss. The unknown investors may not buy stocks even there is not enough compensation to the risk of information. Based on this, the issuers are awaiting the two kinds of investors to buy the stocks to have enough demand on the market. The issuers compensate the information risk which the unknown investors bear, so that the unknown investors can be attracted to buy the stocks. Later, some researchers also make further study based on the Rock's model, Betty and Ritter [5] found that there is a significant relationship between IPO Underpricing rate and the uncertainty of the stocks' intrinsic value on the issuing company. Benveniste L. M. and P. A. Spindt [6], Benveniste and Wilhelm [8] put forward the Information Extraction Theorem and established a Book-building Model. They thought that the issuers underprice the stock in order to make the known investors inform the others to know the prospect and quality about the issue firm once they know the relative information about the company. Habib and Ljungqvist [9] put forward that the issuers tried to formulate the costs level which affected underpricing reasonably to have the smallest profit loss based on the Rock's model.

(3) The information asymmetry between the investors and the issue firm The earliest study about this is Rock [4]. He pointed out the issuers hoped that the investors on the market could recognize the stocks' real intrinsic value through underpricing the stock, because the issuers know more about the real value of the stock to be issued than the investors on the market. Later, Welch [10] put forward Hypothesis of Market feedback which used to be after many scholars. He divided the issue company into two categories: the company of better quality which has the sufficient funds for turnover when issuing and the company of worse quality which has the relatively little money. The frontier one could issue the stock for shares because of the sufficient funds. In the early of issuing new shares, the frontier one attracts

the investors to buy the stocks through underpricing because this makes the investors know about the prospect of the company until enough knowing. Then they can estimate the inherent value of the stock to find that it is higher than the issue price. Afterwards, the issued company again issues the stock with the higher price in the later stage to compensate the loss in the initial stage. However, the later one only issues once in consideration of the little funds. Based on the model of Welch's, Allen and Faulhaber [11] thought that the issued firm is willing to issue with the higher price, so that the investors regard this company as having the better prospect and invest this company. Crinblatt and Hwang [12] also thought so, because only the company with better quality can remedy the cost in the first stage. Brennan and Franks [13] again pointed out the investors which are attracted by underpricing are more than in the anticipation, so the investors on the large-scale institution would not affect the right of management and the right is still controlled by the issuers. Even though many researchers think the important factor of underpricing is the information asymmetry, later others put forward the opposite ideas. Cartter and Manaster [14] posed that the quality of the invested bank is in negative relation with IPO underpricing through studying in the medium institution. Koh and Walter [16] hold that the firms attract more investors to participate in the market through underpricing through studying the popular effect. Ritter [17] found that the information asymmetry might not be the estimate factor of underpricing. Later, a great deal of researchers analyzed underpricing from other factors. Jenkinson, Jones [19] considered the distributed field of the initial issued stock to receive that the information asymmetry have little effect on underpricing. Loughran and Ritter [20] considered the prospect of the company and the psychological accounting. The traditional financial theories are almost studied from the market information. In recent years, some researchers come to study the effect of investor sentiment. Ljungqvist, Nanda, Singh [21] proved there is also IPO underpricing from the aspects to consider investor sentiment under the assumption of market participants' information symmetry, when the price is used to compensate the risk of the underwriters that the hot market will end ahead of time and the issued shares are to delay. Rajan and Servaes [22] found that irrational investors make the stock closing price is much higher than the intrinsic value of the stock and IPO underpricing increases obviously through studying the optimistic investors. Derrien [23] also thought the stock closing price is too high because of investor optimism.

From Table 1.1, we can also see that the rate of IPO underpricing in Chinese stock market is much higher than many other countries or regions. Many researchers analyzed this serious phenomenon and mainly considered from the distribution, pricing and Chinese special institutional factors.

(1) Issuing Mode. Chinese stock mostly issued in the way of underwriters underwriting. There is the time difference between the released stocks in the primary market and the existed stock in the secondary market, which will produce certain effect on the IPO price. Su, Fleisher [24] explained the IPO underpricing based on the waiting time and signal theory and thought that the company underprice shares in order to allow investors to gain value signal. They found the underpricing rate is high up to 949% through studying the newly issued 308 shares in A stock market from 1990 to 1995. Su [25] discovered that IPO underpricing in Chinese A stock market can be explained by the "winners' curse" and "signal model" through empiric in general which were proposed by the foreign researchers. Li [26] posed that the size of underpricing was not affected by the influence of the system in the company, but had negative correlation with the issue price of the stock in the primary market and the number

of distribution and decreased with the increase of issue price and issue number. Mok, Hui [27] found the average IPO underpricing rate of 101 companies of initial public offering is 289.2% from 1990 to 1993 when making Shanghai A stock market as an example and the reason is that the stocks in the secondary market need a waiting time to issue. Chen, Firth, Kim [28] thought IPO underpricing in domestic stock market may have resulted from the following aspects: the issuers do not have enough experience, the investors do not have either, the waiting time between issue in the initial market and list in the secondary market. Chan, Wei, Wang [29] put forward that the more the waiting time and IPO quota system, the more IPO underpricing and decreased as they decreased. Fei Y., Li H. and Liu Y. [30] discussed the reasons of IPO underpricing from the initial market and the secondary market through combining the IPO theory knowledge with the market behavior.

(2) Pricing Model. At present, the pricing modes of Chinese IPO underpricing include inquiry system, control earnings etc. Wang Haifeng et al [31] thought the underpricing rate determined by inquiry system is lower than by control earning, which is because the issuers and the underwriters can more accurately determine the IPO price in the process of inquiry. Yang Jijun, Zhao Changwen and Yang Dan [32] found the IPO indirect cost in the inquiry system reduced namely IPO underpricing rate decreased through the comparison in the point of the issuing company (means the size of underpricing is also an indirect cost). Ma Junlu and Liu Jia [33] proved the correlation of the underpricing rate and the pricing mode through comparing the stocks under the inquiry system.

(3) Institutional Factors. The institutional factors in China include the split share structure, government regulation and so on, which all enlarge the rate of underpricing. Xiao Shuguang and Jiang Shuncaai [35] analyzed the effect on IPO underpricing from the aspect of institutional changes. Liu Yuhui and Xiongpeng [36] posed the basic factor that the domestic IPO underpricing is higher than external is the institutional factor in China. Only make improved measures in the aspect of the split share structure and government regulation, can the serious phenomenon of Chinese IPO underpricing change. Boji and Zhou Xiaohua [37] divided IPO underpricing into two parts: the underpricing in the market and in the rules. And the underpricing in the market can be used to explain the underpricing in various countries or areas, whereas the underpricing in the rules is resulted in the special rules in China, and the reform of the split share structure can bring in defined effects and changes. Throughout the research situation at home and abroad, it is not difficult to find that the researchers at abroad mainly explain underpricing in the information asymmetry and few studies have considered the whole process of issuance. However, the researchers at home are almost based on the external theoretical model, and assume IPO underpricing factors from empiricism, then prove the validity of the results through analyzing the empiric. In 2005, there are institutional changes in Chinese stock market, which is the split share structure reform. The reform has executed for 6 years, but a lot of Chinese listed company stocks are still belong to non-circulation shares and the reform still have not been solved completely. A lot of scholars have discussed the effects of equity division reform on IPO underpricing in response to this change which is mostly empirical analysis and not given a complete model to prove the specific effects to IPO underpricing. This paper builds the IPO underpricing models based on the equity division reform from the angle of quantitative analysis and provides the optimization strategy of IPO, and tests the feasibility and the stability of the model through the empirical analysis.

The organization of this paper is as follows.

In Section 1, we briefly introduce IPO and compare the rate of underpricing in some countries or regions, and describes the research status of the IPO underpricing at home and abroad.

In Section 2, we firstly offer the assumptions of the underpricing model and establish the relationship between the price, quantity of the non-tradable shares and the tradable stocks, and distribute the whole process of the circulation stock into different stages, then the emotional and rational investors value the intrinsic value of each of the stock based on their own ideas before the stocks are listed.

In third Section, we mainly establish the underpricing model of two stages issued stocks based on the equity division. Firstly, we establish the issuer profit maximization model when directly sale to know the number of issued stocks in two stages of the secondary market. Then, we consider the participation constraint when the single underwriter underwrites all the circulating stocks. The individual underwriter in the market can be divided into two categories: the underwriters do not hold non-tradable shares and hold parts. And we determine the total circulation and the IPO price of the circulation stock accordingly draw the underpricing formula. Finally, we study the effects to the underpricing on the number of the issued shares, the ratio of the price and the quantity between the non-tradable and the tradable stocks, the long-term gains.

The forth Section extents from two aspects based on the third section: the two stages extended into multiple stages; the single underwriters changed into a plurality of underwriters. Then, we offer the optimal issuance strategy the underwriters should take in different situations.

In fifth Section, according to the conclusions in the front chapters, we validate the model in the third section according to the data of the initial public offering companies of A shares on Shanghai stock exchange market from 2000 to 2010. The results posed here show that the proposed theoretical model is applicable and has a certain reference value.

The solved problems and innovation of this paper are as follows

(1) Compared with the previous literature based on the market information, this article considers all the IPO distribution process from the aspect of the investor sentiment in the hot market. And we put forward the optimal issuance strategy of the issuers and the underwriters is going to issue the stock in stages to obtain higher profit based on the share of the market.

(2) This paper considers the underwriters as the investors participating in the market compared with the previous articles which only consider the underwriters as the agency. Based on the equity division, respectively discuss the underwriters do not hold non-tradable shares and hold portion, obtain the theory model of the IPO underpricing, study the effects to the underpricing on the various factors of the equity division, and consider the market effect after the reform of the equity division.

(3) The two stages extended into multiple stages and the single underwriters changed into a plurality of underwriters, then we obtain the IPO underpricing.

(4) This paper tests the feasibility and the correctness of the offered models and the theoretical results according to the empirical analysis.

2 Preliminaries

2.1 The Model Assumption Conditions

In terms of issuances and market participants, the model admits the following several assumptions:

Hypothesis 1: The issued company only issues the initial public offerings of shares in hot markets within the short term.

Hypothesis 2: The emotions of all the participants in the IPO market and the secondary market are divided into two kinds, namely the perceptual and the rational. The perceptual investors think the next phase of the market will continue to be the status, in which the market is the hot market, the next phase will also be the hot market and the opposite is also. However, the rational investor is not so, they can estimate rationally the fundamental value of the asset. As is now the hot market, the rational investor will think the next stage may be the hot market, also may no longer be the hot market. Similarly, as is now a bear market, the rational investors are not convinced that the next phase is still a bear market.

Hypothesis 3: The retail investor is sensible, and in hot market, they always estimate over the assets. Other participates (the issuers, the underwriters, the institutional investors) are all rational, and they can make unbiased appraisal assets according to the future of the company.

Hypothesis 4: There is not the private or asymmetric information in the market. All the information on the market is public and the participants all know each other's view. That is, in the IPO stock valuations, the perceptual and rational investors all know each other's valuation size, but they all insist on their own point of view, and their own view is more correct.

Hypothesis 5: All the participants in the market (issuers, underwriters, institutional investors, retail investors) are risk neutral.

This paper is to study IPO underpricing in consideration of the investors' mood, but this is different from other researchers' analysis to finance. The scholars who have deep research to behavioral finance generally applied it to the asset pricing, such as D. Hirshleifer [38] researched income predictability and the phenomenon of the equity premium. This paper mainly researches what kind of distribution strategy the issuers and underwriters should adopt when there are both the sensible and the rational investors. In the past there have been a lot of assets in the pricing, combining with the factor of the participants' mood. T. Odean [39] put forward that people usually have a consciousness of feeling good about themselves, and whether the fact is correct or not, they always believe their own point is right. Such emotions can be called overconfidence. A lot of practice has proved that overconfidence makes people always attribute their like work achievement to their own ability, and few people hope that other factors on the role of the work achievement are more than their own merits. Danie, Hirshleifer, Subrahmanyam [40] once made a careful study in the psychology. In the hot market, the mood of the investors has a strong influence on the stock price. As time goes by, the demand in the market gradually reduces, the hot market gradually fade, the emotions of the investors gradually abate, and the long-term performance of the stock may not be very good. Loughran, Ritter, Rydqvist [41] also divided the issued process into several time stages, and this is related to the emotional change of the perceptual investors. Lee, Shleifer, Thaler [42] also put forward most of the issued firm public offer the stock

only when the stock is in great demand, the stock is very good, and the spirit is very high. Therefore, the assumptions above are reasonable.

Due to the existence of equity division, there are many non-circulation stocks in Chinese A-share market. Assuming all the circulation shares are publicly and completely sold out in the secondary market. All the non-tradable shares are completely sold out in relatively lower prices and people can transfer agreement privately.

Let denote the total circulation of the stock by Q , of which, the number of non-tradable shares is denoted by Q_{FLT} , the number of the overall tradable shares is Q_{LT} , so $Q = Q_{FLT} + Q_{LT}$. Let the ratio of non-tradable shares and tradable shares be θ , that is, $\frac{Q_{FLT}}{Q_{LT}} = \theta$, it is obvious that $Q = (1 + \theta)Q_{LT}$.

The model posed in this paper should be divided the whole issued process of IPO stock in primary and secondary market into four stages $t = 0, 1, 2, 3$.

(1) When $t = 0$, the stock will be issued in the primary market and the issuers will determine the quantity and the price of the current stock. $t = 1, 2, 3$, three stages are the stage where all the current stock is issued in the secondary market.

(2) When $t = 1$, the current stock began to public offer in the secondary market. This stage is the certain bull market in which the stock is sold like hot cakes. In short-term issue, $t = 1$ can also be the first issued day of the circulated stock in the secondary market. After the issuance in the secondary market, that is the back of the stage, the market may continue to the bull market according to the expectation of the perceptual investors, may also no longer be the hot market, but whatever the case is, the prices in the bull market will eventually have a clear end.

(3) When $t = 2$, this is a period of time from $t = 1$ to the clear end of the hot market, that is, the market is uncertain in this stage, and it may be like the expectation of the perceptual investors for the hot market, but can also dodge quickly and is no longer a hot market.

(4) When $t = 3$, this is the time of the end of the hot market when all the stocks issued in the secondary market. In this time the perceptual and rational investors all know the stock price is down to the intrinsic value points, and the valuation of the stock is the intrinsic value.

Denote the IPO price by P_0 , the prices of the current stock in $t = 1, 2, 3$, respectively, by P_1, P_2, P_3 , and write $\frac{P_{FLT}}{P_1} = \gamma$.

Since the market is uncertain in the stage of $t = 2$, the market may continue to bullish on the front stage, and may also be the weakened market and no longer a hot market. Note μ as the probability of the end of the hot market at $t = 2$. Two kinds of participants have different opinions to the stock value, in essence, are holding the different opinions to the size of μ . The perceptual investors think that the hot market will certainly continue to $t = 2$, namely the hot market will end in the probability of $\mu = 0$ at $t = 2$. However, the rational investors do not think so, they think that the hot market will end in the probability of $\mu > 0$ at $t = 2$.

2.2 The Valuations of Stocks Before Listing

There is no bonus allocation and discount in the process of stock issuance. Set V_t as the intrinsic value of the stock at t . Before IPO listing in the secondary market, the intrinsic value of the each stock is through rational forecast to the business prospect by the rational investors, denoted as $V_L = E(V_t)$. If there are only rational investors in the market, V_L is the intrinsic value of each IPO which is first listed in the secondary market (namely $t = 1$). Since

$t = 3$ is the clear end time of the hot market, this time the stock price is down to intrinsic value point, namely $P_3 = V_L$.

For convenience, the hypotheses in this paper that the perceptual investors are constrained by budget and their appraisal of the IPO are quite different from the rational investors' view before IPO listing in the secondary market. Because their budget demand of IPO is the current stock in the secondary market, the total demand curve to IPO can be recorded as:

$$Q_{LT}^d = \frac{V_L + a}{b} - \frac{V_G}{b}.$$

The perceptual investors' valuation to each of the IPO intrinsic value is:

$$V_G = V_L + a - bQ_{LT}^d \tag{2.2.1}$$

where a is constant and is the overestimated part caused by the perceptual investors optimism than the rational investors, also is the protruding part to each IPO appraisal by the perceptual investors when $Q_{LT}^d = 0$. b is the slope of the function, and indicates the opposite change of V_G caused by Q_{LT}^d when it changes a unit.

When $V_G = V_L$, then $Q_{LT}^d = \frac{a}{b}$, one can define $\bar{Q} = \frac{a}{b}$ as the general optimal demand;

For any $Q_{LT}^d < \bar{Q}$, there is $V_G > V_L$, that is, for each IPO, the valuations of the perceptual investors is higher than the rational investors';

For any $Q_{LT}^d > \bar{Q}$, there is $V_G < V_L$, that is, for each IPO, the valuations of the perceptual investors is lower than the rational investors';

When $Q_{LT}^d = \bar{Q}$, there is $V_G = V_L$, that is, for each IPO, the valuations of the perceptual investors is equal to the rational investors'.

Since there is no private or asymmetric information, the perceptual investors and the rational investors all know each other's valuations of IPO. But they estimate the intrinsic value of each stock according to their own respective idea. This hypothesis is not to say all the perceptual investors take participate in the market in $t = 1$, so that we can easily set up a dynamic model in which the hot market may be ended before $t = 2$. If the market continues to be the good market (the hot market) on the front stage in $t = 2$, there will be other perceptual investors who see the good prices to participate in this market. Therefore, the demand of the perceptual investors is in two sessions of $t = 1$ and $t = 2$. The perceptual investors value the stock before buying the stock, and they can buy the stock whose price is lower than their valuations. Because of their perceptual mood, they think the next stage will surely be a hot market, and they will value to each stock based on the forecasts of the IPO demand in $t = 2$. The perceptual investors' forecast of the demand includes not only the perceptual demand in $t = 1$, but also the demand of all the perceptual investors in $t = 2$. The demand in the whole process of the issue will bring a certain change to V_G in type (2.2.1).

3 IPO Underpricing Models Based on Two Stage Issuances With Splitting Share Structure Reform

3.1 IPO Sold Directly

To sell IPO shares directly is that the issuers sell their stocks to the investors and do not pass any of the underwriters. Although the way of this issue that the company in China's stock market take is very little, the issuers can also cooperate with the underwriters through

a combination of the optimal issue strategy when direct marketing. To obtain the issue price of the IPO shares in $t = 0$ and their listed price in $t = 1$, it is necessary to calculate the transaction price of each IPO shares in $t = 2$. If the bull market continued until $t = 2$ based on the expectation of the emotional investors, then the price will be determined by the valuation of the emotional investors to the intrinsic value of each stock in formula (2.2.1). If the bull market is ahead of the end, the trading price of each stock in $t = 2$ will be the valuation of the rational investors to the intrinsic value of each stock before listing, i.e., the intrinsic value of a clear end of the bull market, that is $P_2 = V_L$. Set the initial public offering circulation of the tradable shares $Q_{LT} < \bar{Q}$, and then there will be an optimal solution of Q_{LT} to maximum the profits of the issuers (see below). In $t = 0$, the total number of the tradable stocks and the non-tradable shares determined by the issuers is Q , of which the number of the tradable shares is $Q_{LT} = \frac{Q}{1+\theta}$, the rational investors and emotional investors in $t = 1$ respectively do the following estimates according to their own point of view to the transaction price of each IPO stock in $t = 2$:

The rational investors: $E_L(P_2) = \mu V_L + (1 - \mu)E_G(P_2)$;

The perceptual investors: $E_G(P_2) = V_L + a - bQ_{LT}^d$.

Since the emotional investors believe that the bull market will certainly continue to $t = 2$, they forecast the demand in $t = 2$ is the total emotional demand at the two time periods of $t = 1, 2$. So their expected price in $t = 2$ is consistent with V_G , that is $E_G(P_2) = V_L + a - bQ_{LT}^d$. Emotional investors' and rational investors' expected price in $t = 2$ is also the price they were willing to buy the stock in $t = 1$, because if they purchase the stocks in accordance with the price of $t = 1$, they believe that they can sell them at this price in $t = 2$, at least without loss. When the market achieved a balanced status, the demand is equal to the issuance, that is $Q_{LT}^d = Q_{LT}$, thus there are

$$\begin{aligned} E_G(P_2) &= V_L + a - bQ_{LT}^d \\ &= V_L + a - bQ_{LT} \\ E_L(P_2) &= \mu V_L + (1 - \mu)E_G(P_2) \\ &= \mu V_L + (1 - \mu)(V_L + a - bQ_{LT}) \\ &= V_L + (1 - \mu)(a - bQ_{LT}). \end{aligned}$$

On the other hand, there is $V_G > V_L$ if $Q_{LT} < \bar{Q}$, that is, $a - bQ_{LT} > 0$, so $E_L(P_2) > V_L$. The expected price of the rational investors in $t = 2$ is higher than its intrinsic value, which is because the expectation of the participants in a rational market (such as the issuer, the underwriters) is that if the market continued to be a hot market in $t = 2$ in accordance with the probability of $(1 - \mu)$, then they will sell the remaining stocks to the perceptual investors.

Let P_1 be the trading price in $t = 1$, for investors, the price they are willing to pay in $t = 1$ is their expected trading price of the stocks in $t = 2$. Let $\bar{Q}_1 \leq \bar{Q}$ be the optimal demand of the emotional investors in $t = 1$. When $q_1 \leq \bar{Q}_1$, the circulation is less than the demand of the emotional investors in $t = 1$, the emotional investors is optimistic to the market and their valuation of each stock is higher than the rational investors', so the issuers sell stocks q_1 to the emotional investors with the price of $E_G(P_2)$ in $t = 1$. When $q_1 > \bar{Q}_1$, the circulation is more than the demand of the emotional investors in $t = 1$, and the emotional investors think the market is not good enough and will not hold too much optimism to the market. The investors would not buy shares in $t = 1$, so the issuers sell stocks q_1 to the rational investors with the price of $E_L(P_2)$ and they sell them to the emotional investors when the hot market

emerge in $t = 2$. That is

$$P_1 = \begin{cases} E_L(P_2) = \mu V_L + (1 - \mu)E_G(P_2) = V_L + (1 - \mu)(a - bQ_{LT}), & \text{if } q_1 > \bar{Q}_1 \\ E_G(P_2) = V_L + a - bQ_{LT}, & \text{if } q_1 \leq \bar{Q}_1 \end{cases} \quad (3.1.1)$$

Suppose the issuer determine the circulation and the price of the IPO shares to get the most profit for their own. Assume that the issuance Q of the stocks is given, and then the number of the tradable shares Q_{LT} is also identified. Recorded q_1 as the sold number of IPO shares on the secondary market in $t = 1$, and q_2 as the sold number of the IPO shares on the secondary market in $t = 2$. Assume the issuer is rational, and his expected price in $t = 2$ is $E_L(P_2)$. For the issuers his proceeds include the proceeds of all the sold tradable shares and non-tradable shares, that is, there exists

$$\begin{aligned} \Psi &= q_1 P_1 + q_2 E_L(P_2) + Q_{FLT} P_{FLT} \\ &= q_1 P_1 + q_2 [\mu V_L + (1 - \mu)E_G(P_2)] + \theta Q_{LT} \gamma P_1. \end{aligned}$$

The issuer's profit function is the part of the earnings over its intrinsic value after the stock issued, that is $\pi = \Psi - V_L Q$. The optimal circulation (q_1^*, q_2^*) in $t = 1, 2$ is the optimal solution of the following constraint problem

$$\begin{cases} \max_{q_1, q_2} \pi = \Psi - V_L Q \\ \quad = q_1 P_1 + q_2 (\mu V_L + (1 - \mu)E_G(P_2)) + \theta Q_{LT} \gamma P_1 - V_L Q \\ \quad = q_1 P_1 + q_2 (\mu V_L + (1 - \mu)E_G(P_2)) + \theta Q_{LT} \gamma P_1 - V_L (1 + \theta) Q_{LT} \\ \text{s.t. } q_1 + q_2 = Q_{LT}. \end{cases}$$

Theorem 3.1.1. Assume that the total amount Q of all the tradable stocks and non-tradable stocks, and the number of the tradable shares $Q_{LT} = \frac{Q}{1+\theta}$ are given, then the optimal circulation (q_1^*, q_2^*) in $t = 1, 2$ is given by

$$(q_1^*, q_2^*) = \begin{cases} (Q_{LT}, 0), & \text{if } Q_{LT} \leq \bar{Q}_1 \\ (\bar{Q}_1, Q_{LT} - \bar{Q}_1), & \text{if } Q_{LT} > \bar{Q}_1 \end{cases} \quad (3.1.2)$$

Proof. By using the formula (3.1.1), we get

1) When $Q_{LT} > \bar{Q}$, a) if $q_1^* > \bar{Q}_1$, then $P_1 = E_L(P_2) = V_L + (1 - \mu)(a - b(q_1 + q_2))$. Let $\hat{q}_1 \hat{=} \bar{Q}_1$, the circulation $q_2 = \hat{q}_2 \hat{=} Q_{LT} - \bar{Q}_1$, in $t = 2$, for $\hat{q}_1 < q_1^*$. It is easy to see that from the above equation P_1 is decreasing with respect to q_1 , that is, $P_1(\hat{q}_1) > P_1(q_1^*)$, and one can also have the following

$$\begin{aligned} \pi &= q_1 P_1 + q_2 E_L(P_2) + \theta \gamma Q_{LT} P_1 - V_L (1 + \theta) Q_{LT} \\ &= q_1 P_1 + q_2 + P_1 + \theta \gamma Q_{LT} P_1 - V_L (1 + \theta) Q_{LT} \\ &= Q_{LT} P_1 + \theta \gamma Q_{LT} P_1 - V_L (1 + \theta) Q_{LT}. \end{aligned}$$

Thus we know that $\pi(\hat{q}_1) > \pi(q_1^*)$.

b) If $q_1^* = \bar{Q}_1$, then $P_1 = E_G(P_2)$. Set $\hat{q}_1 \hat{=} \bar{Q}_1$, the circulation $q_2 = \hat{q}_2 = Q_{LT} - \bar{Q}_1$, in $t = 2$, for $\hat{q}_1 > q_1^*$. By a direct computation, it is easy to show that there holds the following

$$\begin{aligned} \pi &= q_1 P_1 + q_2 E_L(P_2) + \theta \gamma Q_{LT} P_1 - V_L (1 + \theta) Q_{LT} \\ &= q_1 E_G(P_2) + q_2 (\mu V_L + (1 - \mu)E_G(P_2)) + \theta \gamma Q_{LT} E_G(P_2) - V_L (1 + \theta) Q_{LT} \\ &= (q_1 + q_2) E_G(P_2) + \mu q_2 V_L - \mu q_2 E_G(P_2) + \theta \gamma Q_{LT} E_G(P_2) - V_L (1 + \theta) Q_{LT} \\ &= Q_{LT} [E_G(P_2) + \theta \gamma E_G(P_2) - V_L (1 + \theta)] - \mu (Q_{LT} - q_1) (E_G(P_2) - V_L). \end{aligned}$$

This shows in this case that π is an increased function of q_1 .

In summary, when $Q_{LT} > \bar{Q}_1$, only when $q_1^* = \hat{q}_1 = \bar{Q}_1$, π is achieved the maximum.

2) When $Q_{LT} \leq \bar{Q}_1$, $P_1 = E_G(P_2)$. Similarly, we know that $\pi = Q_{LT}[E_G(P_2) + \theta\gamma E_G(P_2) - V_L(1 + \theta)] - \mu(Q_{LT} - q_1)(E_G(P_2) - V_L)$. π is an increased function of q_1 . That is to say, π is achieved the maximum when $q_1^* = Q_{LT}$. The proof of Theorem 3.1.1 is completed. \square

From Theorem 3.1.1, it can be seen that the issuer who sold all the IPO shares in the market at once will not have more profit than that with two phases issued. In order that the buyers in $t = 1$ are all the emotional investors, the issuers can control the circulation $q_1 \leq \bar{Q}_1$ to make the supply be less than the demand of emotional investors. Intuitively, it can be easily seen that if the number of the total circulation $Q_{LT} < \bar{Q}_1$, the optimal distribution strategy of the issuers is to sell all the tradable shares to the perceptual investors at a higher price at $t = 1$, such that the circulation in $t = 2$ is $q_2^* = 0$. If the issuers can sell the stocks to the emotional investors at a higher price in $t = 1$, whereas deliberately delayed until $t = 2$, they will not have the above advantages of the price. If the bull market ends ahead of time in $t = 2$, the sold-delay will make the issuer face a greater market cash flow risk. If the number of total circulation of shares $Q_{LT} > \bar{Q}_1$, the optimal distribution strategy of the issuer is to sell parts of the stocks to the emotional investors in $t = 1$ by using their emotional mood to make such a misconception that the bull market will continue to $t = 2$, and then sell the remaining shares to the emotional investors in the next phase at a higher price.

3.2 Individual Underwrite IPO Shares

Underwriting the IPO shares is the phenomenon that the issuing company sells all the stocks which will be issued and listed to issue listed to the securities intermediaries (namely the underwriters) in the initial market with a certain price, then the underwriters trade these stocks in the secondary market. The securities intermediaries underwriting IPO shares can be divided into two categories: underwriters do not hold the non-tradable shares and the underwriters hold parts of the non-tradable shares. Here IPO underpricing problem are discussed based on the two cases.

3.2.1 Underpricing When Underwriters No-holding Non-tradable Shares

Assume that prohibit buying and selling stocks among the underwriters, and allowsthe underwriters to sell the stocks in stages. If the underwriter does not hold non-tradable shares, only underwriting all of the outstanding shares, once the demand function of the emotional investor is created, the underwriters buy all the tradable stocks Q_{LT} from the issuer with the price of P_0 in the primary market, and then sell q_1, q_2 , in $t = 1, t = 2$. When the stock is sold, the problem of the underwriters in the secondary market is the same with the previous problem of the issuer's direct marketing. The revenue function of the underwriters: $\Psi' = q_1P_1 + q_2E_L(P_2)$. When the underwriters determine the circulation in two time periods of $t = 1, t = 2$, the problem is given as follows

$$\begin{cases} \max_{q_1, q_2} \Psi' = q_1P_1 + q_2E_L(P_2) \\ \text{s.t. } q_1 + q_2 = Q_{LT} \end{cases}$$

Theorem 3.2.1. *The optimal circulation of the underwriters in $t = 1, t = 2$ is given by*

$$(q_1^*, q_2^*) = \begin{cases} (Q_{LT}, 0), & \text{if } Q_{LT} \leq \bar{Q}_1 \\ (\bar{Q}_1, Q_{LT} - \bar{Q}_1), & \text{if } Q_{LT} > \bar{Q}_1 \end{cases} \quad (3.2.1)$$

Proof. According to Formula 3.1.1, one gets the following

1) In case $Q_{LT} > \bar{Q}_1$.

a) If $q_1^* > \bar{Q}_1$, then $P_1 = E_L(P_2) = V_L + (1 - \mu)[a - b(q_1 + q_2)]$. Set $\hat{q}_1 \hat{=} \bar{Q}_1$, the circulation in $t = 2$ $q_2 = \hat{q}_2 = Q_{LT} - \bar{Q}_1$ for $\hat{q}_1 < q_1^*$. It can be seen from the above equation that P_1 is a decrease function with respect to q_1 , that is, $P_1(\hat{q}_1) > P_1(q_1^*)$. On the other hand, there is $\Psi' = q - 1P_1 + q - 2E_L(P_2) = (q_1 + q_2)P_1 = Q_{LT}P_1$, then we know $\Psi'(\hat{q}_1) > \Psi'(q_1^*)$.

b) If $q_1^* < \bar{Q}_1$, then $P_1 = E_G(P_2)$. Set $\hat{q}_1 = \bar{Q}_1$, the circulation in $t = 2$ $q_2 \hat{=} Q_{LT} - \bar{Q}_1$ for $\hat{q}_1 > q_1^*$. At the same time, there holds

$$\begin{aligned} \Psi' &= q_1 P_1 + q_2 E_L(P_2) \\ &= q_1 E_G(P_2) + q_2 [\mu V_L + (1 - \mu) E_G(P_2)] \\ &= q_1 E_G(P_2) + q_2 \mu V_L + q_2 E_G(P_2) - q_2 \mu E_G(P_2) \\ &= (q_1 + q_2) E_G(P_2) + \mu q_2 V_L - \mu q_2 E_G(P_2) \\ &= Q_{LT} E_G(P_2) + \mu q_2 (V_L - E_G(P_2)) \\ &= Q_{LT} E_G(P_2) + \mu (Q_{LT} - q_1) (V_L - E_G(P_2)) \\ &= Q_{LT} [E_G(P_2) - \mu (Q_{LT} - q_1) (E_G(P_2) - V_L)]. \end{aligned}$$

This implies that Ψ' is an increased function of q_1 . That is to say, $\Psi'(q_1^*) < \Psi'(\hat{q}_1)$.

In summary, when $Q_{LT} > \bar{Q}_1$, and only if $q_1^* = \hat{q}_1 = \bar{Q}_1$, Ψ' is achieved the maximum.

2) When $Q_{LT} \leq \bar{Q}_1$, $P_1 = E_G(P_2)$. Similarly, we get $\Psi' = Q_{LT} E_G(P_2) - \mu [(Q_{LT} - q_1) (E_G(P_2) - V_L)]$ and Ψ' is an increased function of q_1 . Thus, we know that Ψ' is achieved the maximum when $q_1^* = Q_{LT}$. The ends the proof of Theorem 3.2.1. \square

Remark 3.2.1. *It is obvious that the optimal distribution strategy of the underwriters is to sell q_1^*, q_2^* , in $t = 1, t = 2$. This strategy also requires that when the number of the circulation shares $Q_{LT} > \bar{Q}_1$, the underwriters must remain q_2^* copies of the outstanding shares until $t = 2$. It can be seen from the payoff function of the underwriters that the underwriters have no motivation to comply with the sales strategy.*

In order to achieve a balanced state, the underwriters will underwrite the IPO shares only when their own profit is not less than 0. If the underwriters purchase all the outstanding shares from the issuer according to the price P_0 of each IPO stock in the primary market, and sell q_1^* with the price P_1 of in $t = 1$ and sold q_2^* with the price $E_L(P_2)$ in $t = 2$, then the underwriter's profit function is $\pi' = q_1^* P_1 + q_2^* E_L(P_2) - Q_{LT} P_0$, where q_1^*, q_2^* are the optimal solution of (3.2.1). $q_1^* P_1 + q_2^* E_L(P_2)$ is the benefits which the underwriters sell stocks in the secondary market in $t = 1, t = 2$ to get. $Q_{LT} P_0$ is the costs that the underwriters buy all the IPO tradable stocks in the primary market.

The constraints the underwriters involved in the market are as follows

$$\begin{aligned} \pi' &= q_1^* P_1 + q_2^* E_L(P_2) - Q_{LT} P_0 \\ &= q_1^* P_1 + q_2^* [\mu V_L + (1 - \mu) E_G(P_2)] - Q_{LT} P_0 \\ &\geq 0. \end{aligned} \quad (3.2.2)$$

As in the previous assumptions, the goal of the issuer is to make the profit function π that the price to sell the stock exceed to its intrinsic value achieves the maximum, coupled with the constraints of the above (3.2.2). The problem that the issuers determine the issue number and issue price is exactly as

$$\begin{cases} \max_{Q, P_0} \pi = Q_{LT}P_0 + Q_{FLT}P_{FLT} - QV_L \\ \text{s.t. } \pi' \geq 0. \end{cases}$$

Lemma 3.2.1. *The constraints the underwriters involved in the market do always exist.*

Proof. If there is no such constraints, then the issue number Q_{LT} and the issue price P_0 of the tradable shares does not have any restrictions. The issuers can be unrestricted to improve Q_{LT} and P_0 to increase their profits, which is conflicted with the existence of the optimal solution to Q_{LT} and P_0 . Therefore, the constraint is always there. \square

Based on Formula (3.2.2), it is known

$$\pi' = q_1^*P_1 + q_2^*[\mu V_L + (1 - \mu)E_G(P_2)] - Q_{LT}P_0 \geq 0.$$

That is

$$q_1^*P_1 + q_2^*[\mu V_L + (1 - \mu)E_G(P_2)] \geq Q_{LT}P_0.$$

The issuer's profit function can be written as

$$\begin{aligned} \max_{Q, P_0} \pi &= Q_{LT}P_0 + Q_{FLT}P_{FLT} - QV_L \\ &= q_1^*P_1 + q_2^*[\mu V_L + (1 - \mu)E_G(P_2)] + \theta\gamma Q_{LT}P_1 - (1 + \theta)Q_{LT}V_L. \end{aligned}$$

where q_1^*, q_2^* are the optimal solution in front equation (3.2.1). $q_1^*P_1 + q_2^*(\mu V_L + (1 - \mu)E_G(P_2))$ is the expected return of the issuers to the IPO shares sold in two phases. $\theta Q_{LT}\gamma P_1$ is the proceeds that the issuers sell all the non-tradable shares. $(1 + \theta)Q_{LT}V_L$ is the intrinsic value of all the tradable shares and non-tradable shares. By Theorem 3.2.1, we can get $q_1^* \leq \bar{Q}_1$, so P_1 is the expected price of the emotional investors in $t = 2$, that is, $P_1 = E_G(P_2)$. Then, we get the issuer's profit function as

$$\begin{aligned} \max_{Q_{LT}} \pi &= q_1^*P_1 + q_2^*(\mu V_L + (1 - \mu)E_G(P_2)) + \theta\gamma Q_{LT}P_1 - (1 + \theta)Q_{LT}V_L \\ &= q_1^*E_G(P_2) + q_2^*(\mu V_L + (1 - \mu)E_G(P_2)) + \theta Q_{LT}\gamma E_G(P_2) - (1 + \theta)Q_{LT}V_L. \end{aligned}$$

where q_1^*, q_2^* are the optimal solution of (3.2.1), which are the function of the circulation Q_{LT} of the outstanding shares.

Theorem 3.2.2. *If there is only one underwriter in the stock-market without holding any non-tradable shares, then the optimal choice of the issuers to the number of the outstanding shares is*

$$Q_{LT}^* = \begin{cases} \frac{\theta(\gamma-1)V_L + Aa - \mu b \bar{Q}_1}{2b(1+\gamma\theta-\mu)}, & \text{if } \bar{Q}_1 < \frac{\theta(\gamma-1)V_L + Aa}{B}, \\ \frac{(1+\gamma\theta)a + \theta(\gamma-1)V_L}{2b(1+\gamma\theta)}, & \text{otherwise} \end{cases}$$

where $A = 1 + \gamma\theta - \mu, B = 2b + 2b\gamma\theta - \mu b$.

The optimal sold quantity q_1^* and q_2^* in $t = 1, t = 2$ are

$$(q_1^*, q_2^*) = \begin{cases} (\bar{Q}_1, \frac{\theta(\gamma-1)V_L + Aa - \mu b \bar{Q}_1}{2b(1+\gamma\theta-\mu)}), & \text{if } \bar{Q}_1 < \frac{\theta(\gamma-1)V_L + Aa}{B}, \\ (\frac{(1+\gamma\theta)a + \theta(\gamma-1)V_L}{2b(1+\gamma\theta)}, 0), & \text{otherwise} \end{cases}$$

The optimal number of tradable shares and non-tradable shares issued by the issuer are as follows

$$Q^* = \begin{cases} (1 + \theta) \frac{\theta(\gamma-1)V_L + Aa - \mu b \bar{Q}_1}{2b(1+\gamma\theta-\mu)}, & \text{if } \bar{Q}_1 < \frac{\theta(\gamma-1)V_L + Aa}{B}, \\ (1 + \theta) \frac{(1+\gamma\theta)a + \theta(\gamma-1)V_L}{2b(1+\gamma\theta)}, & \text{otherwise} \end{cases}$$

The price of the IPO stocks is

$$P_0 = \frac{Q_{LT}P_1 + \mu q_2^* V_L - \mu q_1^* P_1}{Q_{LT}}.$$

The IPO underpricing is

$$R = P_1 - P_0 = \frac{\mu q_2^* (P_1 - V_L)}{Q_{LT}}.$$

Proof. 1) When $Q_{LT} > \bar{Q}_1$, there hold the following $q_1^* = \bar{Q}_1, q_2^* = Q_{LT} - \bar{Q}_1$, and

$$\begin{aligned} \max_{Q_{LT}} \pi &= q_1^* P_1 + q_2^* (\mu V_L + (1 - \mu) E_G(P_2)) + \theta Q_{LT} \gamma P_1 - (1 + \theta) Q_{LT} V_L \\ &= q_1^* E_G(P_2) + q_2^* [\mu V_L + (1 - \mu) E_G(P_2)] + \theta Q_{LT} \gamma E_G(P_2) - (1 + \theta) Q_{LT} V_L \\ &= q_1^* E_G(P_2) + q_2^* \mu V_L + q_2^* (1 - \mu) E_G(P_2) + \theta Q_{LT} \gamma E_G(P_2) - (1 + \theta) Q_{LT} V_L \\ &= (q_1^* + q_2^*) E_G(P_2) + \mu q_2^* (V_L - E_G(P_2)) + \theta \gamma Q_{LT} E_G(P_2) - (1 + \theta) Q_{LT} V_L \\ &= (1 + \theta \gamma) Q_{LT} (V_L + a - b Q_{LT}) + \mu (Q_{LT} - \bar{Q}_1) (b Q_{LT} - a) - (1 + \theta) Q_{LT} V_L. \end{aligned}$$

By a direct computation, we get

$$\begin{aligned} \frac{d\pi}{dQ_{LT}} &= (1 + \theta \gamma) (V_L + a - 2b Q_{LT}) + \mu (b Q_{LT} - a + b Q_{LT} - b \bar{Q}_1) - (1 + \theta) V_L \\ &= 2b Q_{LT} (\mu - 1 - \theta \gamma) + (\gamma - 1) \theta V_L + (1 + \theta \gamma - \mu) a - \mu b \bar{Q}_1. \end{aligned}$$

Set $\frac{d\pi}{dQ_{LT}} = 0$, one gets

$$Q_{LT}^* = \frac{\theta(\gamma-1)V_L + (1+\gamma\theta-\mu)a - \mu b \bar{Q}_1}{2b(1+\gamma\theta-\mu)}.$$

In this case, there is $Q_{LT}^* > \bar{Q}_1$, that is, $Q_{LT}^* = \frac{\theta(\gamma-1)V_L + (1+\gamma\theta-\mu)a - \mu b \bar{Q}_1}{2b(1+\gamma\theta-\mu)} > \bar{Q}_1$, thus we have

$$\bar{Q}_1 < \frac{\theta(\gamma-1)V_L + (1+\gamma\theta-\mu)a}{2b + 2b\gamma\theta - b\mu}.$$

2) When $Q_{LT} \leq \bar{Q}_1$, we have $q_1^* = Q_{LT}, q_2^* = 0$, and

$$\begin{aligned} \max_{Q_{LT}} \pi &= q_1^* P_1 + q_2^* (\mu V_L + (1 - \mu) E_G(P_2)) + \theta Q_{LT} \gamma P_1 - (1 + \theta) Q_{LT} V_L \\ &= (1 + \theta \gamma) Q_{LT} (V_L + a - b Q_{LT}) - (1 + \theta) Q_{LT} V_L. \end{aligned}$$

Similarly we arrive at

$$\begin{aligned} \frac{d\pi}{dQ_{LT}} &= (1 + \theta\gamma)(V_L + a - 2bQ_{LT}) - (1 + \theta)V_L \\ &= -2bQ_{LT}(1 + \theta\gamma) + (1 + \theta\gamma)a + \theta(\gamma - 1)V_L. \end{aligned}$$

Put $\frac{d\pi}{dQ_{LT}} = 0$, we obtain

$$Q_{LT}^* = \frac{\theta(\gamma - 1)V_L + (1 + \gamma\theta)a}{2b(1 + \gamma\theta)}.$$

When the number of total issued shares and the number of the outstanding shares have been identified, again calculate the IPO price. Then, we get

$$\pi' = q_1^*P_1 + q_2^*[\mu V_L + (1 - \mu)E_G(P_2)] - Q_{LT}P_0 \geq 0$$

The equality holds if the profits of the issuers to achieve the maximum. In this case, one has

$$P_0 = \frac{Q_{LT}P_1 + \mu q_2^*V_L - \mu q_2^*P_1}{Q_{LT}}.$$

The IPO underpricing is

$$R = P_1 - P_0 = \frac{\mu q_2^*(P_1 - V_L)}{Q_{LT}}.$$

This implies that Theorem 3.2.2 is proved. □

If you only consider the sentiment of the investors, then if $Q_{LT} > \bar{Q}_1$, i.e., $\bar{Q}_1 < \frac{\theta(\gamma-1)V_L+(1+\theta\gamma-\mu)a-\mu b}{2b+2b\gamma\theta-\mu b}$, the issuer gets the most profits when $q_2^* > 0$. In stock-underwriting, the issuer hopes that the underwriters hold part of the outstanding shares until $t = 2$. At $t = 2$, if the market continues to be the hot market, then the underwriter can sell the stock to the perceptual investors at higher prices at this time; if the market is no longer a hot market, then the stock price will be reduced to its rational intrinsic value V_L . For the underwriters, they need to take the risk of the end of the hot market and the reduced stock, so when they buy the tradable stocks from the issuer, the price that is the issue price at the first time needs a discount, related to the first trading price in the secondary market, $P_0 < P_1$. When \bar{Q}_1 is large enough, in this model, with $\bar{Q}_1 \geq \frac{\theta(\gamma-1)V_L+(1+\gamma\theta-\mu)a-\mu b}{2b+2b\theta\gamma-\mu b}$, $q_2^* = 0$. In $t = 1$, the underwriters can sell all the outstanding shares at the higher valuation of the emotional investors to the emotional investors. In this case, the underwriters will no longer bear the risk of the termination of the hot market in $t = 2$. For the issuers, the underpricing will only reduce their own profits, and no benefits are available. It does not exist underpricing, $P_0 = P_1$.

4 The Expanded Model Based On Two Stage Issuances and On A Single Underwriter Of Split Share Structure Reforms

4.1 The Expanded Multiple-Term Models

In order to simplify the model, we here assume that the underwriters do not hold the non-tradable shares. Suppose that the demands of the perceptual investors can be expanded

to more time $0, 1, 2, \dots, T$. But as the market is gradually faded, the need of the perceptual investors is less and less gradually. Especially, note: $Q_{t+1} = kQ_t, 0 < k < 1$. Other conditions are same with the model hypothesis of the first two stages issuance, the hot market in each next period will end in probability μ and is to be continued with the probability $(1 - \mu)$ for the following hot market. The general circulation of the tradable stock is big enough to meet the needs of the perceptual investors to the stock in T stage. The best strategy to sell is issuing all the tradable stocks into T periods, to make the perceptual investors participate in the market in each time period, until all the current stocks have been sold or the hot market has a clear end time. The valuation of the perceptual investors to each stock before listing is still $V_L + a - bQ_{LT}$. The general circulation of the tradable stock is

$$Q_{LT} = Q_1 + Q_2 + \dots + Q_T = \frac{1 - k^T}{1 - k} Q_1.$$

The total circulation of the tradable stocks and the non-tradable stocks in the issuing company is

$$Q = Q_{LT} + Q_{FLT} = (1 + \theta) \frac{1 - k^T}{1 - k} Q_1.$$

The underwriters buy current stocks Q_{LT} with the price P_0 in initial market, and sell Q_1 in $t = 1$. If the hot market is still in next period, then sell Q_2 at $t = 2$, in turn so on, until all the current stocks are sold. The profit function of the issuer is

$$\pi = Q_{LT}(P_0 - V_L) + \theta Q_{LT}(\gamma P_1 - V_L).$$

The participated constraint condition of the underwriters is as follows

$$Q_1 P_1 + Q_2 E_L(P_2) + Q_3 E_L(P_3) + \dots + Q_T E_L(P_T) - Q_{LT} P_0 \geq 0.$$

Then, we arrive at the problem below

$$\begin{cases} \max \pi = Q_{LT}(P_0 - V_L) + \theta Q_{LT}(\gamma P_1 - V_L) \\ s.t. Q_1 P_1 + Q_2 E_L(P_2) + Q_3 E_L(P_3) + \dots + Q_T E_L(P_T) - Q_{LT} P_0 \geq 0. \end{cases}$$

Similar to the model of the front two-period, when the market has enough circulations, the price $E_L(P_s)$ that the underwriter sell the stocks in $t = s(1 \leq s \leq T - 1d)$ is the valuation $E_G(P_{s+1})$ that the perceptual investors estimate in next time $s + 1$. So we get

$$E_G(P_2) = P_1, E_G(P_3) = E_L(P_2), \dots, E_G(P_T) = E_L(P_{T-1})$$

$$\begin{aligned} E_L(P_2) - V_L &= \mu V_L + (1 - \mu)E_G(P_2) - V_L \\ &= (1 - \mu)(P_1 - V_L) \\ E_L(P_3) - V_L &= \mu V_L + (1 - \mu)E_G(P_3) - V_L \\ &= (1 - \mu)^2(P_1 - V_L) \\ &\dots \\ E_L(P_T) - V_L &= (1 - \mu)^{T-1}(P_1 - V_L). \end{aligned}$$

When the profit of the issuer achieves the maximum, in this case, there holds

$$Q_1 P_1 + Q_2 E_L(P_2) + Q_3 E_L(P_3) + \dots + Q_T E_L(P_T) - Q_{LT} P_0 = 0$$

By a direct computation, we have

$$\begin{aligned} Q_{LT}(P_0 - V_L) &= Q_{LT}P_0 - Q_{LT}V_L \\ &= Q_1P_1 + Q_2E_L(P_2) + \dots + Q_TE_L(P_T) - (Q_1 + Q_2 + \dots + Q_T)V_L \\ &= Q_1(P_1 - V_L) + kQ_1(1 - \mu)(P_1 - V_L) + \dots + k^{T-1}(1 - \mu)^{T-1}(P_1 - V_L) \\ &= Q_1(P_1 - V_L)\frac{1 - k^T(1 - \mu)^T}{1 - k(1 - \mu)}. \end{aligned}$$

Since $P_1 = E_G(P_2) = V_L + a - bQ_{LT}$, $Q_{LT} = \frac{1-k^T}{1-k}Q_1$, then we arrive at

$$\begin{aligned} Q_{LT}(P_0 - V_L) &= Q_1\left(a - b\frac{1 - k^T}{1 - k}Q_1\right)\frac{1 - k^T(1 - \mu)^T}{1 - k(1 - \mu)}. \\ \pi &= Q_{LT}(P_0 - V_L) + \theta Q_{LT}(\gamma P_1 - V_L) \\ &= Q_1\left(a - b\frac{1 - k^T}{1 - k}Q_1\right)\frac{1 - k^T(1 - \mu)^T}{1 - k(1 - \mu)} + \theta\frac{1 - k^T}{1 - k}Q_1[\gamma(V_L + a - b\frac{1 - k^T}{1 - k}Q_1) - V_L]. \end{aligned}$$

Let $\frac{d\pi}{dT} = 0$, we obtain $T^*(\gamma, \theta)$. Thus we know that the optimal amount of the current stock issuance is

$$Q_{LT} = \frac{1 - k^{T^*}}{1 - k}Q_1,$$

and the total circulation of the tradable stocks and the non-tradable stocks in the issuing company is

$$Q = (1 + \theta)\frac{1 - k^{T^*}}{1 - k}Q_1.$$

The price of the stock in $t = 1$ is

$$P_1 = E_G(P_2) = V_L + a - b\frac{1 - k^{T^*}}{1 - k}Q_1.$$

The price of IPO stock is

$$P_0 = V_L + \left(a - b\frac{1 - k^{T^*}}{1 - k}Q_1\right)\frac{1 - k^{T^*}(1 - \mu)^{T^*}}{1 - k(1 - \mu)}\frac{1 - k}{1 - k^{T^*}}.$$

The suppression price of IPO stock is

$$R = P_1 - P_0 = \left(a - b\frac{1 - k^{T^*}}{1 - k}Q_1\right)\left(1 - \frac{1 - k^{T^*}(1 - \mu)^{T^*}}{1 - k(1 - \mu)}\frac{1 - k}{1 - k^{T^*}}\right).$$

Similarly, we can get the relation between the ratio γ of the IPO price with the price of the non-tradable and tradable stocks and the ratio θ of the number of the non-tradable stocks with the tradable stocks.

Assume that the probability for a hot market in each period is $(1 - \mu)$, and the probability of the termination for the hot market is μ , then the length of the expected time of the hot market is

$$E(T) = 1 \cdot \mu + 2(1 - \mu)\mu + 3(1 - \mu)^2\mu + \dots.$$

Theorem 4.1.1. *When the circulation of the current stock is big enough to meet the perceptual demand in every period, then the length of the expected time of the hot market is $E(T) = \frac{1}{\mu}$.*

Proof. By using $E(T) = 1 \cdot \mu + 2(1 - \mu)\mu + 3(1 - \mu)^2\mu + \dots$ and a direct computation, it is not hard to show that $E(T) = \frac{1}{\mu}$. □

Remark 4.1.1. *Theorem 4.1.1 implies that the length of the hot market is less than 2 years in the sense of expected times.*

4.2 The Model Expands To Multiple Underwriters

If there are $N(N > 1)$ -underwriters simultaneously underwriting the IPO shares on the market, then each of the underwriters is free to choose its own issuance strategy on the stock. Regarding all of the underwriters as a unit, then its optimal strategy is consistent with only one underwriter's, and is still holding some shares until $t = 2$. However, for each of the underwriters is a separate individual, he is more willing to privately sell all the stocks in $t = 1$ to avoid the risk of the termination of the hot market in $t = 2$. To prohibit privately selling all the stock in $t = 1$, which is the phenomenon of affecting the profits of the issuers, the issuers must take a mechanism of reward or punishment, which will award the underwriters who comply with the commitments, and punish the underwriters who are contrary to the commitments. Assume the award of the issues to the underwriters for the price of each IPO stock is lower ϵ , and then the issue price of the IPO shares in the initial market is

$$\hat{P}_0 = P_0 - \epsilon.$$

where P_0 is the IPO price that the underwriters do not hold the non-tradable shares. If the expectations of the additional profit which the average of the each underwriters are expected to abide by the commitment is $\lambda\epsilon$, where λ has a relation with the number of the stocks the underwriters maintain to $t = 2$ and the market-behavior in $t = 2$. If the underwriters comply with the commitments, they can sell q_1^* -shares with the price P_1 in $t = 1$, and expect the trading price of the stock in $t = 2$ is $E_L(P_2)$. If the underwriters break their promises, all of the stocks will be sold in $t = 1$, then the average extra profits each underwriter obtained is

$$\frac{q_2}{N}(P_1 - E_L(P_2)).$$

Therefore, in order to make the underwriters comply with their promises, it must have the following constraints

$$\lambda\epsilon \geq \frac{q_2}{N}(P_1 - E_L(P_2)).$$

The revenue function the underwriters complying with the commitments is

$$\pi' = q_1(P_1 - V_L) + q_2[(\mu V_L + (1 - \mu)E_G(P_2)) - V_L].$$

The problem the underwriters determine the circulation in two periods turns into

$$\begin{cases} \max_{q_1, q_2} \pi' = q_1(P_1 - V_L) + q_2[(\mu V_L + (1 - \mu)E_G(P_2)) - V_L] \\ \text{s.t. } \lambda\epsilon \geq \frac{q_2}{N}(P_1 - E_L(P_2)) \end{cases}$$

Theorem 4.2.1. *If there are N -underwriters involved in the market, and the underwriters do not hold the non-tradable shares, then the optimal circulations of the underwriters in $t = 1, t = 2$ are*

$$\begin{cases} q_1^* = \bar{Q}_1 \\ q_2^* = \frac{\mu(a-b\bar{Q}_1) - \sqrt{\mu^2(a-b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b} \end{cases}$$

Proof. The income function of the underwriters, in fact, by a direct computation, is

$$\begin{aligned} \pi' &= q_1(P_1 - V_L) + q_2[(\mu V_L + (1 - \mu)E_G(P_2)) - V_L] \\ &= (q_1 + q_2(1 - \mu))(a - b(q_1 + q_2)). \end{aligned}$$

The constraints the underwriters involved in the market are simplified as

$$\lambda\epsilon \geq \frac{\mu q_2}{N}(a - b(q_1 + q_2)).$$

So, the equality holds if the revenue function of the underwriters takes the maximum, that is, we get

$$\lambda\epsilon = \frac{\mu q_2}{N}(a - b(q_1 + q_2)). \quad (4.2.1)$$

Similarly, let the optimal demand of the emotional investors in $t = 1$ be \bar{Q}_1 , then it is easy to see that the optimal circulation in $t = 1$ is \bar{Q}_1 , i.e. $q_1^* = \bar{Q}_1$. Substituting it into (4.2.1), we have

$$\begin{aligned} \lambda\epsilon &= \frac{\mu q_2}{N}(a - b(\bar{Q}_1 + q_2)) \\ q_2^* &= \frac{\mu(a - b\bar{Q}_1) \pm \sqrt{\mu^2(a - b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b}. \end{aligned}$$

If $q_2^* = \frac{\mu(a-b\bar{Q}_1) + \sqrt{\mu^2(a-b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b}$, one substitutes it into (4.2.1), and gets

$$\begin{aligned} a - b(q_1^* + q_2^*) &= a - b\left[\bar{Q}_1 + \frac{\mu(a - b\bar{Q}_1) + \sqrt{\mu^2(a - b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b}\right] \\ &= a - b\frac{\mu(a + b\bar{Q}_1) + \sqrt{\mu^2(a - b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b} \\ &= -\frac{\mu b\bar{Q}_1 + \sqrt{\mu^2(a - b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b} \\ &< 0. \end{aligned}$$

This implies that $\lambda\epsilon = \frac{\mu q_2}{N}(a - b(q_1 + q_2)) < 0$. This is contradicted with the hypothesis!. In other words, we arrive at

$$q_2^* = \frac{\mu(a - b\bar{Q}_1) - \sqrt{\mu^2(a - b\bar{Q}_1)^2 - 4\mu bN\lambda\epsilon}}{2\mu b}.$$

This ends the proof of Theorem 4.2.1. □

Remark 4.2.1. From Theorem 4.2.1, we know, when more underwriters involved in the market, the optimal strategy of the issuers is also to issue the stocks in phases. This also calculates the issue number of the tradable stocks, the issue price of IPO and the underpricing size of IPO.

5 Empirical Analysis

5.1 Empirical Preparations

5.1.1 Data Sources

Because Chinese split share structure reform is into effect in 2005, we select the data of the several years before and after the reform to compare and analyse. The title is to select the data of 436 initial public offering companies in A-share market of Shanghai Stock Exchange from January 1, 2000 to December 31, 2010. Here collects the number of the tradable shares and the non-tradable shares, the first day closing price of the listed tradable stocks, the listing net assets per share, the issue price of per share of all the company's initial public offering and so on. Individual data has large differences with the routine situations, so exclude it in the regression analysis.

5.1.2 Hypothesis

The theoretical model in Section 3 discussed the relationship between the underpricing of the initial public offering stock and the circulation of the tradable stocks, the long-term gains of the stocks, the ratio of the price and the number of the non-tradable shares and the tradable shares when the split share structure exists. There are the following assumptions

1. Before the split share structure reform, because of the existence of a large number of non-tradable shares, the IPO underwriting has a negative relationship with the ratio γ of the price of the non-tradable shares and the tradable shares, and has an uncertain relationship with the ratio θ of the number of the non-tradable shares and the tradable shares; at the same time, has a negative relation with the circulation of the tradable shares, and has a positive ratio with the long-term gains ($P_1 - V_L$) of the stocks.

2. After the split share structure reform, the non-tradable shares become the tradable shares and the domestic market is similar to the foreign market. The IPO underpricing has a relation with two factors, and has a negative relation with the circulation of the tradable shares, and has a positive ratio with the long-term gains ($P_1 - V_L$) of the stocks.

5.1.3 The Analysis of The Sample

According to the data of every year from different initial public offering companies in Shanghai Stock Exchange, we can calculate the average ratio of the IPO underpricing in every year. The average underpricing ratio in 11 years is as shown in Table 5.1.1. From Table 5.1.1, before the split share structure reform, the underpricing rate is high to 131.6% in A-share market. However, since the split share structure reform is into effect in 2005, the previous non-tradable shares are now all changed into the tradable shares, and the underpricing rate is largely decreased, the average underpricing rate in 5 years is descended to 57.2%. The underpricing rate before and after the split share structure reform is identical with the research result of the present researchers. Among which, the underpricing rate in

the first two years after the split share structure reform (2006 and 2007) is unusual, which may be because of the non-thorough or non-overall achievement of the systems at the beginning of the reform and parts of the non-tradable shares becoming limit-sold shares. However, the reform has a great influence on the underpricing rate on the whole, which also explains the set of data can be used to verify the effect of the different factors of this reform to the underpricing rate.

Table 5.1.1 The average IPO underpricing rate in every year from 2000 to 2010.

year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
aur	1.524	2.256	1.832	0.732	0.713	0.85	0.354	1.162	0.499	0.559	0.286
ipon	86	75	67	66	61	3	14	24	5	9	26

where "aur" denotes for the average underpricing rate, "ipon" denotes for the number of IPO companies. The number of the initial public offering companies in Shanghai Stock Exchange from 2000 to 2010 is 436.

By using Table 5.1.1, we know that the average IPO underpricing rate from 2000 to 2004 is 1.316; The average IPO underpricing rate from 2006 to 2010 is 0.572.

5.1.4 The Description of Variables

When we design the variability, the Ministry of Finances in China ruled the exchanged price of the non-tradable stocks is not less than the net asset per share, so most of the trading price is equal to the net assets per share when the issuers sell the non-tradable shares. Therefore, the net assets per share are as the trading price of the non-tradable share.

1. The circulation of the tradable stocks and the IPO underpricing. Typically, the more the circulation of the tradable stocks, the greater the size of the company, it also will be more attractive to the investors on the market. Therefore, each department will make more stringent regulatory measures to the companies, to ensure the fair and validity of the stock issued. Then, the more the circulation of the tradable stocks, the lower the IPO underpricing.

2. The long-term gains and the IPO underpricing. The greater the long-term gains ($P_1 - V_L$) of the stocks, the higher the closing price P_1 of the first day or the smaller the intrinsic value V_L of the stocks. And the higher the closing price P_1 of the first day, the bigger the IPO underpricing $P_1 - P_0$. If the stock's intrinsic value V_L is smaller, the trading price of the non-tradable shares will be smaller, the interests of the holders with the tradable shares will have greater losses, the IPO underpricing will be greater. So the greater the long-term gains, the greater the IPO underpricing.

3. The ratio of the price of the non-tradable shares with the tradable shares and the IPO underpricing. The higher the price of the non-tradable shares, the closer to the price of the tradable shares, the more relatively fair the holders of the tradable shares, the smaller the loss of the interests of the investors to buy the tradable shares, and the lower the IPO underpricing. Therefore, the higher the ratio of the price of the non-tradable shares with the tradable shares, the smaller the IPO underpricing.

4. The ratio of the number of the non-tradable shares with the tradable shares and the IPO underpricing. Intuitively, if the ratio of the number decreases, the rate of IPO underpricing should be reduced. However, it can be seen in the average rate of IPO underpricing in 2007, the IPO underpricing can not be reduced only to reduce the number of the non-tradable shares. The essence of the affect of the non-tradable shares on the underpricing is the effect

of the trading price. The split share structure reform not only reduced the number of the non-tradable shares, but also improved the trading price of the non-tradable shares. Therefore, the ratio of the number of the non-tradable shares with the tradable shares has no uncertain correlation.

5.2 The Analysis of Regression Models

5.2.1 The Analysis of Models Before the Reform

According to the situation prior to the reform, note the underpricing as the dependent variable, and the four factors before the reform as independent variables, one can establish the following regression model:

$$R = a_1 \ln(LTSL) + b_1 CQSY + c_1 JGB + d_1 SLB + const.$$

where $R = P_1 - P_0$ is the IPO underpricing, P_0 is the price with which the issuers in initial market sell the stocks to the underwriters (that is the IPO price), P_1 is the closing price for the first day issuance of the underwriters in the secondary market. $\ln(LTSL)$ is the natural logarithm of the number of all the tradable stocks of IPO; $CQSY$ is the long-term gains of the stocks, and that is the difference $(P_1 - P_0)$ for the first day closing price of the IPO stock in the secondary market and its intrinsic value. JGB is the ratio of the trading price of the non-tradable stocks with the first day closing price of the tradable stocks in the secondary market. SLB is the ratio of the number of the IPO non-tradable shares and the tradable stocks.

The data of the model before the reform is to select the data of the initial public offering companies in A-share market of Shanghai Stock Exchange from January 1, 2000 to December 31, 2005. Regression analysis using SPSS, the results of the empirical tests is in the following Table 5.2.1.

Table 5.2.1 The Regression Results of Tests Before The Reform

Variables	Coefficients	Standard Derivations	T-Value	P-Value
const	3.195	1.929	1.656	0.099
$\ln(LTSL)$	-0.635	0.215	-2.954	0.003
CQSY	0.737	0.021	34.939	0.000
JGB	-4.474	0.773	-5.787	0.000
SLB	0.077	0.035	2.172	0.031
R	0.917	R-square	0.84	
F-value	464.798	F-P-value	0.000	
Observations	357			

It can be seen from Table 5.2.1, $R = 0.917$, this indicates that the underpricing has a more obvious linear relationship with the four factors before the reform. The coefficient of determination R -square is 0.839, namely, there are about 83.9% of the IPO underpricing of the listed companies can be explained by four factors of the regression model before the share reform, which identifies the pre-share reform regression model has better explain.

The observation value of F -statistics is 464.798, the probability of significantly is 0.000, that the probability of the establishment of the hypothesis of the test " H_0 : the regression coefficients $B = 0$ " is 0.000, that is, one should refuse the null hypothesis, show that the

underpricing has a significant linear relationship with the four factors before the reform, we can create such a linear model.

We can also get the following conclusions from the above coefficients: the coefficient of the natural logarithm of the number of the tradable stocks is $a_1 = -0.635 < 0$, which verifies in Assumption 1 the underpricing has a negative correlation with the circulation of the tradable stocks. The coefficient of the long-term gains is $b_1 = 0.737 > 0$, which verifies in Assumption 1 the underpricing has a positive correlation with the long-term gains. The coefficient of the ratio of the price is $c_1 = -4.474 < 0$, which verifies in Assumption 1 the underpricing has a negative correlation with the ratio of the price. The probability P of the first three t -statistics is far smaller than 0.05, which showed that the first three regression coefficients are significant. But the coefficient of the ratio of the number is $d_1 = 0.077 > 0$, and P value is bigger, which shows the underpricing has no certain correlation with the ratio of the number, and verifies in Assumption 1 the underpricing has uncertain relation with the ratio of the number of the non-tradable shares with the tradable shares. In summary, Assumption 1 is verified.

5.2.2 The Analysis of Models After The Reform

According to the situation after the reform, we denote the underpricing as the dependent variable, and the two factors after the reform as independent variables, and establish the following reformed regression model:

$$R = a_2 \ln(LTSL) + b_2 CQSY + const.$$

The data of the model after the reform is to select the data from the initial public offering companies in A-share market of Shanghai Stock Exchange from January 1, 2006 to December 31, 2010. Regression analysis using SPSS, the results of the empirical tests are in the following Table 5.2.2.

Table 5.2.2 The Regression Results of Tests After The Reform

Variables	Coefficients	Standard Derivations	T-Value	P-Value
const	-5.517	3.481	-1.585	0.117
$\ln(LTSL)$	0.443	0.292	1.516	0.134
CQSY	0.479	0.037	12.861	0.000
R	0.834	R-square	0.696	
F-value	85.860	F-P-value	0.000	
Observations	77			

It can be seen from Table 5.2.2, $R = 0.834$, it indicates that the underpricing has a more obvious linear relationship with the two factors after the reform. The coefficient of determination R -square is 0.696, namely there are about 69.6% of the IPO underpricing of the listed companies can be explained by two factors of the regression model after the share reform, which identifies the regression model after the share reform has better explain.

The observation value of F -statistics is 85.860, the probability of significantly is 0.000, that the probability of the establishment of the hypothesis of the test " H_0 : the regression coefficients $B = 0$ " is 0.000, that is, one should refuse the null hypothesis, show that the underpricing has a significant linear relationship with the two factors after the reform, we can create such a linear model.

We can also get the following conclusions from the above coefficients: the coefficient of the natural logarithm of the number of the tradable stocks is $a_2 = 0.443 > 0$, but the probability P of the t -statistics is bigger, which is because the number of the initial public offering companies in Shanghai Stock Exchange from 2006 to 2010 is small to cause the sample size is too small. The coefficient of the long-term gains is $b_2 = 0.479 > 0$, which verifies the underpricing has a positive correlation with the long-term gains. The probability P of the t -statistics is far smaller than 0.05, which shows that the regression coefficient is significant. Then Assumption 2 is verified.

5.3 A Brief Analysis of The Empirical Results

Taken by the empirical data, as well as the regression model results before and after the share reform, we have the following conclusions:

1. The implementation of the reform in 2005 has an indeed great deal of impact on the underpricing of China's stock market. From the data selected from the empirical in A-share market of Shanghai Stock Exchange listed company from January 1, 2000 to December 31, 2010, it can be seen that the average underpricing rate after the reform is down to 57.2% from 131.6% before the reform, indicating that the split share reform has boosted China's stock market matures, and reduced the rate of China's IPO underpricing.

2. When the split share exists, the underpricing of the stocks in the listed companies from 2000 to 2005 of the evidence selected has a negative correlation with the ratio of the price of the non-tradable shares and the tradable shares; and has no certain correlation with the ratio of the number of the non-tradable shares and tradable shares. The increases of the price of the non-tradable shares will be able to effectively reduce the rate of IPO underpricing in China's listed companies, and just decrease the number of non-tradable shares and change it into tradable shares or restricted shares, this does not necessarily reduce IPO underpricing in China's stock market.

3. After the split share structure reform, consistent with the circulated foreign stock market, the underpricing of the listed companies stocks from 2006 to 2010 of the evidence selected has relation with the number of the tradable stocks and the long-term gains of the stocks. However, after the share reform, the average underpricing rate in China is 57.2%, and compared with the level of the developed countries' average rate of underpricing of 15%, there is a big gap, which may be because the split share reform has so far failed to completely solve, China's government regulation and other institutional factors.

By the empirical results, the split share's key impact on IPO underpricing is the prices of the non-tradable shares. The split share structure reform will not only make most of the non-tradable shares change into the tradable stocks or restricted shares, but also improve the trading price of the non-tradable shares or the restricted shares. Through a combination of both, China's stock market becomes further mature. With the full implementation of this reform, almost all non-tradable stocks will be converted into the tradable shares, that is, the trading price of the previous non-tradable shares is consistent with the tradable shares, it can be seen from the model, which will greatly reduce the IPO underpricing in China's stock market.

The share reform has the effect of long-term strategic significance and practical economic and social effect to regulate China's securities market system, improve the system of the market system, improve market efficiency of investment and financing behavior, build an

open, just, fair and efficient, modern securities market .

It can be said that the split share reform has only just begun, and still has a long way to go in the future. There are many problems which need to ascertain the nature. In this paper, it gives some conclusions of the economic significance by a more rational analysis. Hope to provide the rational policy recommendations to the decision makers.

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