

# Option pricing under risk-exogenous measures in a fractional jump-diffusion market

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**Abstract** In this paper we introduce the so-called risk-exogenous measure and study the price of exogenous risks based on a fractional jump-diffusion financial market model. The option price equation indicates that the evaluation of risk-exogenous is consistent with that of the classical neutral risk. An empirical example shows that the risk-exogenous valuation is more suitable for practical financial markets by comparing the error between actual price of stocks and the price computed from BS formula and the option price equation under risk-exogenous measures respectively.

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**Keywords** risk-exogenous measure; fractional jump-diffusion model; fractional Girsanov theorem; option pricing

**2010 Mathematics Subject Classification** 60H15; 60G22; 91G80

## 1 Introduction

The study of option price has been an active field over the past several decades. It is well known that one deduces via the standard geometric Brownian motion the famous Black-Scholes model [1]. While numerous empirical researches show that fluctuations in stock prices have the properties of fat tail and long memory, which can not be reflected in a geometric Brownian motion. Fractional Brownian motion has been considered as a suitable tool in financial models because it not only allows for fat tails, but also allows for long-range dependence property, see [2],[3],[4].

On the other hands, the stock prices may jump discontinuously due to the influence of some significant information in practical financial markets. So in this paper a fractional jump-diffusion model is adopted to have a full consideration of all these actual factors. Based on the fractional jump-diffusion model, plenty of works focus on option pricing problems. For example, Xue and Sun [5] discuss European option under fractional jump-diffusion Ornstein-Uhlenbeck model and Xue et al. [6] obtain the pricing formula of European option under fractional jump-diffusion financial market model with stochastic interest rate. Some empirical analyses show that the option prices obtained in the fractional jump-diffusion model are much closer to the actual market prices but still have some errors. This may be caused by some risks from external markets and is easy to be ignored.

As we know, the dynamic of asset price process is the reflection of market information to some extent. While some other information from external markets may also affect stock price, such as supply and demand of firm, currencies and prices policy, international trade factors as well as emergencies, which are called exogenous risks. For example, Pearce et al. [7] study the relationship between stock prices and economic news. Rigobon et al. [8] show the effect of monetary policy on stock market. These researches mainly focus on qualitative descriptions and seldom give the quantitative expression. So how to measure the exogenous risks and what are the effects of these risks on the price of underlying risky assets and contingent claims are meaningful questions. It's worth noting that emergencies, that is some significant information, are often characterized by jump terms reflected in equations of stock price. So in this paper we consider some other exogenous risks except for emergencies.

An outline of this paper is as follows. In section 2 a fractional jump-diffusion market model is given and a generalized fractional Ito formula is derived. In section 3 we study the price of exogenous risks by fractional Girsanov measure transformation and introduce the concept of risk-exogenous measure. In section 4 we discuss risk-exogenous valuation. In section 5 an empirical example is given to show that the advantage of risk-exogenous valuation.

## 2 Fractional jump-diffusion market model

Given a probability space  $(\Omega, \mathcal{F}^H, \mathcal{F}_t^H, P)$  and a standard fractional Brownian motion  $\{B_t^H, 0 \leq t \leq T\}$  with  $dB_t^H = \varepsilon\sqrt{dt^{2H}}$  where  $\varepsilon$  follows the standard normal distribution  $N(0, 1)$ . The  $\sigma$ -algebra generated by the fractional Brownian motion is denoted as  $\mathcal{F}_t^H$ , i.e.,  $\mathcal{F}_t^H = \sigma\{B_s^H, 0 \leq s \leq t\}$  and  $\mathcal{F}_T^H = \mathcal{F}^H$ .

Consider the following model: the dynamic of risky asset price process satisfies the following stochastic differential equation

$$dS_t = S_t\{(\mu - \lambda\theta)dt + \sigma dB_t^H + Y_t dN_t\} \tag{2.1}$$

where  $\mu$  and  $\sigma$  are constants and  $\{N_t, 0 \leq t \leq T\}$  is the Poisson process with intensity  $\lambda$ , and the size of the  $i^{th}$  jump is  $Y_i$  with  $Y_0 = 0$  and  $Y_i > -1, i = 1, 2, \dots$ . Assume  $\{Y_i, i \geq 1\}$  is a sequence of independent identically distributed random variables with  $Y_i \sim N(\theta, \delta^2)$  under the probability measure  $P$ . Assume  $\{B_t^H, 0 \leq t \leq T\}$ ,  $\{N_t, 0 \leq t \leq T\}$  and  $\{Y_i, i > 1\}$  are mutually independent. On the other hand, there exists a risk-free money account satisfying

$$dD_t = rD_t dt \tag{2.2}$$

where  $r$  is constant representing risk-free rate and  $D_0 = 1$ . The following lemma can be regarded as a generalized fractional Ito formula.

**Lemma 2.1.** *If the risky asset  $S_t$  satisfies fractional jump-diffusion equation (2.1) and risk-free asset satisfies the equation (2.2), let  $f(t, S_t)$  is a function with continuous partial derivatives  $\frac{\partial f}{\partial t}$ ,  $\frac{\partial f}{\partial S}$  and  $\frac{\partial^2 f}{\partial S^2}$ . Then the following formula holds*

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}S(\mu - \lambda\theta)dt + H\sigma^2t^{2H-1}S^2\frac{\partial^2 f}{\partial S^2}dt + \frac{\partial f}{\partial S}S\sigma dB_t^H + [f(t, (1 + Y)S) - f(t, S)]dN_t. \tag{2.3}$$

Proof. For convenience, we omit the subscript  $t$ . From (2.1) we have

$$\Delta S = S(\mu - \lambda\theta)\Delta t + S\sigma\Delta B^H + SY\Delta N. \tag{2.4}$$

Then

$$\begin{aligned} (\Delta S)^2 &= S^2(\mu - \lambda\theta)^2(\Delta t)^2 + S^2\sigma^2(\Delta B^H)^2 + S^2Y^2(\Delta N)^2 + 2(\mu - \lambda\theta)\sigma S^2\Delta t\Delta B^H \\ &\quad + 2(\mu - \lambda\theta)YS^2\Delta t\Delta N + 2\sigma YS^2\Delta B^H\Delta N \\ &= S^2\sigma^2(\Delta B^H)^2 + S^2Y^2(\Delta N)^2 + o(\Delta t). \end{aligned} \tag{2.5}$$

As for the classical Brownian motion  $\{B_t, t \geq 0\}$ , we know  $(\Delta B)^2 \rightarrow dt$  when  $\Delta t \rightarrow 0$ . For fractional Brownian motion we have the following statements. Since

$$\begin{aligned} Var((\Delta B^H)^2) &= E((\Delta B^H)^4) - (E((\Delta B^H)^2))^2 \\ &= E((\Delta B^H)^4) - (Var(\Delta B^H) + (E(\Delta B^H))^2)^2 \\ &= E(\varepsilon^4(\Delta t^{2H})^2) - (\Delta t^{2H})^2 \\ &= (\Delta t^{2H})^2(E(\varepsilon^4) - 1) \\ &= o(\Delta t). \end{aligned} \tag{2.6}$$

It indicates that when  $\Delta t \rightarrow 0$ ,  $(\Delta B^H)^2$  is a deterministic function with respect to  $t$ , so

$$(\Delta B^H)^2 \rightarrow E((dB^H)^2) = dt^{2H} = 2Ht^{2H-1}dt. \tag{2.7}$$

By Taylor Expansion for  $f(t, S)$ , it holds

$$\Delta f = \frac{\partial f}{\partial S}\Delta S + \frac{\partial f}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}(\Delta S)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial S\partial t}\Delta S\Delta t + \frac{1}{2}\frac{\partial^2 f}{\partial t^2}\Delta t^2 + o((\Delta S)^2 + \Delta t^2). \tag{2.8}$$

It is easy to see that  $\Delta S\Delta t$ ,  $(\Delta t)^2$  and  $o((\Delta S)^2 + \Delta t^2)$  are all  $o(\Delta t)$ . Then combining (2.4) and (2.5), it has

$$\begin{aligned} \Delta f &= \frac{\partial f}{\partial S}S(\mu - \lambda\theta)\Delta t + \frac{\partial f}{\partial S}S\sigma\Delta B^H + \frac{\partial f}{\partial S}SY\Delta N + \frac{\partial f}{\partial t}\Delta t \\ &\quad + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}S^2\sigma^2(\Delta B^H)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}S^2Y^2(\Delta N)^2 + o(\Delta t). \end{aligned} \tag{2.9}$$

Let  $\Delta t \rightarrow 0$ , since

$$\frac{\partial f}{\partial S}SY\Delta N + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}S^2Y^2(\Delta N)^2 = [f(t, (1 + Y)S) - f(t, S)]dN \tag{2.10}$$

then (2.9) can be written as

$$\begin{aligned} df &= \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}S(\mu - \lambda\theta)dt + \frac{\partial f}{\partial S}S\sigma\Delta B^H + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}S^2\sigma^22Ht^{2H-1}dt \\ &\quad + [f(t, (1 + Y)S) - f(t, S)]dN \\ &= \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}S(\mu - \lambda\theta)dt + H\sigma^2t^{2H-1}S^2\frac{\partial^2 f}{\partial S^2}dt + \frac{\partial f}{\partial S}S\sigma dB^H \\ &\quad + [f(t, (1 + Y)S) - f(t, S)]dN. \end{aligned} \tag{2.11}$$

Then the proof is finished. □

### 3 The price of exogenous risks

Consider the market consisting of one risk-free money account  $D_t$  and one risky asset  $S_t$ . let  $\alpha_t$  and  $\beta_t$  be the amounts of money account and risky asset respectively, held at time  $t$ . The value of this portfolio at time  $t$  is given by

$$V_t = \alpha_t D_t + \beta_t S_t. \tag{3.1}$$

Assume the portfolio is self-financing, that is

$$\begin{aligned} dV_t &= \alpha_t dD_t + \beta_t dS_t \\ &= r\alpha_t D_t dt + \beta_t S_t (\mu - \lambda\theta) dt + \beta_t S_t \sigma dB_t^H + \beta_t S_t Y_t dN_t \\ &= r(V_t - \beta_t S_t) dt + \beta_t S_t (\mu - \lambda\theta) dt + \beta_t S_t \sigma dB_t^H + \beta_t S_t Y_t dN_t \\ &= rV_t dt + \beta_t S_t (\mu - r - \lambda\theta) dt + \beta_t S_t \sigma dB_t^H + \beta_t S_t Y_t dN_t \\ &= rV_t dt + \beta_t S_t \sigma \left( \frac{\mu - r + \bar{\lambda}\bar{\theta} - \lambda\theta}{\sigma} dt + dB_t^H \right) + \beta_t S_t (Y_t dN_t - \bar{\lambda}\bar{\theta} dt) \\ &= rV_t dt + \beta_t S_t \sigma dW_t^H + \beta_t S_t (Y_t dN_t - \bar{\lambda}\bar{\theta} dt) \end{aligned} \tag{3.2}$$

where

$$\xi = \frac{\mu - r + \bar{\lambda}\bar{\theta} - \lambda\theta}{\sigma} \tag{3.3}$$

and

$$W_t^H = \xi t + B_t^H. \tag{3.4}$$

By Fractional Girsanov Theorem [9] there exists probability measure  $Q \sim P$  on the space  $(\Omega, \mathcal{F}^H)$  such that  $W_t^H$  is a fractional Brownian motion with respect to measure  $Q$  and  $Q$  is the equivalent quasi-martingale measure of  $P$ . Under this measure transformation, the poisson process  $\{N_t, t \geq 0\}$  have intensity  $\bar{\lambda}$  and expected jump size  $E_Q(Y) = \bar{\theta}$ .

It is not hard to see that the dynamic of the risky asset price process  $S_t$  under  $Q$  is given by the following SDE:

$$dS_t = S_t \{ (r - \bar{\lambda}\bar{\theta}) dt + \sigma dW_t^H + Y_t dN_t \}. \tag{3.5}$$

In general,  $Q$  is called risk-neutral measure. And  $\xi$  in (3.3) consists of two sections: the price of market risks  $\frac{\mu-r}{\sigma}$  and the price of jump risks  $\frac{\bar{\lambda}\bar{\theta}-\lambda\theta}{\sigma}$ .

As we known, except for market risks and jump risks, some other exogenous risks can not be ignored from the variability of external market environment including supply and demand of firm, currencies and prices policy, international trade factors and so on. Here we assume that every contingent claim in the market has exogenous risks and these exogenous risks are proportional to the price of risky asset  $S_t$ .

**Definition 3.1.** *The exogenous risks  $G_t$  in a time interval  $dt$  is defined by*

$$dG_t = \eta S_t dt \tag{3.6}$$

where  $\eta$  is a constant representing a rate of exogenous risks.

The exogenous risks can be considered an additional/less capital required in a time interval  $dt$ , that is

$$\begin{aligned}
 dV_t &= \alpha_t dD_t + \beta_t (dS_t + dG_t) \\
 &= r\alpha_t D_t dt + \beta_t S_t (\mu - \lambda\theta + \eta) dt + \beta_t S_t \sigma dB_t^H + \beta_t S_t Y_t dN_t \\
 &= r(V_t - \beta_t S_t) dt + \beta_t S_t (\mu - \lambda\theta + \eta) dt + \beta_t S_t \sigma dB_t^H + \beta_t S_t Y_t dN_t \\
 &= rV_t dt + \beta_t S_t (\mu - r + \eta - \lambda\theta) dt + \beta_t S_t \sigma dB_t^H + \beta_t S_t Y_t dN_t \\
 &= rV_t dt + \beta_t S_t \sigma \left( \frac{\mu - r + \bar{\lambda}\bar{\theta} - \lambda\theta + \eta + \tilde{\lambda}\tilde{\theta} - \bar{\lambda}\bar{\theta}}{\sigma} dt + dB_t^H \right) \\
 &\quad + \beta_t S_t (Y_t dN_t - \tilde{\lambda}\tilde{\theta} dt) \\
 &= rV_t dt + \beta_t S_t \sigma (dW_t^H + \frac{\eta + \tilde{\lambda}\tilde{\theta} - \bar{\lambda}\bar{\theta}}{\sigma} dt) + \beta_t S_t (Y_t dN_t - \tilde{\lambda}\tilde{\theta} dt) \\
 &= rV_t dt + \beta_t S_t \sigma dZ_t^H + \beta_t S_t (Y_t dN_t - \tilde{\lambda}\tilde{\theta} dt)
 \end{aligned} \tag{3.7}$$

where

$$\gamma = \frac{\eta + \tilde{\lambda}\tilde{\theta} - \bar{\lambda}\bar{\theta}}{\sigma} \tag{3.8}$$

and

$$Z_t^H = \gamma t + W_t^H. \tag{3.9}$$

Fractional Girsanov Formula [9] means that there exists probability measure  $R \sim Q$  on the space  $(\Omega, \mathcal{F}^H)$  such that  $Z_t^H$  is a fractional Brownian motion and  $R$  is the equivalent quasi-martingale measure of  $Q$ . Under this new measure  $R$ , the Poisson process  $\{N_t, t \geq 0\}$  have intensity  $\tilde{\lambda}$  and expected jump size  $E_R(Y) = \tilde{\theta}$ . Then we can get the dynamic of the risky asset price process  $S_t$  under  $R$  is given by the following SDE:

$$dS_t = S_t \{ (r - \eta - \tilde{\lambda}\tilde{\theta}) dt + \sigma dZ_t^H + Y_t dN_t \}. \tag{3.10}$$

Here we call the measure  $R$  risk-exogenous measure since it is subjected to exogenous risks  $G_t$ . And  $\gamma$  in (3.8) consists of two sections: the price of exogenous risks  $\frac{\eta}{\sigma}$  and the price of jump risks  $\frac{\tilde{\lambda}\tilde{\theta} - \bar{\lambda}\bar{\theta}}{\sigma}$ .

## 4 Option valuation under risk-exogenous measures

In this section we discuss the effect of exogenous risks on contingent claims valuation. The following theorem indicates that the risk-exogenous valuation is consistent with the risk-neutral valuation in form. Here we consider a contingent claim with the payoff  $f_T = f(T, S_T)$  at time  $T$ . Let  $f_T$  is bounded and  $f_T \in L^2(\Omega, \mathcal{F}^H)$ .

**Theorem 4.1.** *The dynamic of risky asset price process follows (3.10). Then the price of the contingent claim  $f_T$  at time  $t$  is given by*

$$f = f(t, S_t) = E_R[e^{-r(T-t)} f_T | \mathcal{F}_t^H] \tag{4.1}$$

where  $E_R[\cdot | \mathcal{F}_t^H]$  refers to the quasi-conditional expectation under the probability measure  $R$ . Thus  $f$  satisfies the following equation

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} S(r - \eta - \tilde{\lambda}\tilde{\theta}) + H\sigma^2 t^{2H-1} S^2 \frac{\partial^2 f}{\partial S^2} + \tilde{\lambda} E_R[f(t, (1+Y)S) - f(t, S)] = rf. \tag{4.2}$$

Proof. From the quasi-martingale pricing theorem in [10], we know any bounded contingent claims  $f_T$  with  $f_T \in L^2(\Omega, \mathcal{F}^H)$  can be priced by the quasi-conditional expectation of their discounted payoff, that is (4.1) holds. From Lemma 2.1 when  $S_t$  follows (3.10) it is easy to see that

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} S(r - \eta - \tilde{\lambda}\tilde{\theta})dt + H\sigma^2 t^{2H-1} S^2 \frac{\partial^2 f}{\partial S^2} dt + \frac{\partial f}{\partial S} S\sigma dZ_t^H + [f(t, (1+Y)S) - f(t, S)]dN_t. \quad (4.3)$$

We can deduce from (4.1) that

$$e^{-rt} f(t, S_t) = E_R[e^{-rT} f(T, S_T) | \mathcal{F}_t^H] \quad (4.4)$$

which implies that  $e^{-rt} f(t, S_t)$  satisfies quasi-martingale property under the measure  $R$ . Then by taking differential for the term  $e^{-rt} f(t, S_t)$  it has

$$\begin{aligned} d(e^{-rt} f(t, S_t)) &= e^{-rt}(-rf dt + df) \\ &= e^{-rt}(-rf dt + \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} S(r - \eta - \tilde{\lambda}\tilde{\theta})dt + H\sigma^2 t^{2H-1} S^2 \frac{\partial^2 f}{\partial S^2} dt \\ &\quad + \tilde{\lambda}E_R[f(t, (1+Y)S) - f(t, S)]dt + \frac{\partial f}{\partial S} S\sigma dZ_t^H \\ &\quad + [f(t, (1+Y)S) - f(t, S)]dN_t - \tilde{\lambda}E_R[f(t, (1+Y)S) - f(t, S)]dt). \end{aligned} \quad (4.5)$$

Rewrite (4.5) as follows

$$d(e^{-rt} f(t, S_t)) = e^{-rt}(A dt + B + C) \quad (4.6)$$

where

$$\begin{aligned} A &= -rf + \frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} S(r - \eta - \tilde{\lambda}\tilde{\theta}) + H\sigma^2 t^{2H-1} S^2 \frac{\partial^2 f}{\partial S^2} \\ &\quad + \tilde{\lambda}E_R[f(t, (1+Y)S) - f(t, S)], \end{aligned} \quad (4.7)$$

$$B = \frac{\partial f}{\partial S} S\sigma dZ_t^H, \quad (4.8)$$

$$C = [f(t, (1+Y)S) - f(t, S)]dN_t - \tilde{\lambda}E_R[f(t, (1+Y)S) - f(t, S)]dt. \quad (4.9)$$

According to the quasi-martingale property of fractional Brownian motion and

$$E_R(C) = 0 \quad (4.10)$$

the coefficient  $A$  of the term  $dt$  must be equal to 0. Then (4.2) is easy to be obtained.  $\square$

## 5 Examples

In this section we give an example to indicate the price computed by risk-exogenous valuation is closer to the actual price compared with the risk-neutral valuation. For the convenience of calculation, here we consider the classic Brownian motion case with  $H = \frac{1}{2}$ . First we know the option pricing equation under the risk-neutral measure  $Q$  is

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} Sr + \frac{1}{2}(\sigma^2 + \bar{\lambda}E_Q[Y^2])S^2 \frac{\partial^2 f}{\partial S^2} = rf. \quad (5.1)$$

According to Theorem 4.1, the option pricing equation under the risk-exogenous measure  $R$  is

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}S(r - \eta) + \frac{1}{2}(\sigma^2 + \tilde{\lambda}E_R[Y^2])S^2 \frac{\partial^2 f}{\partial S^2} = rf. \tag{5.2}$$

Given the special parameters, then we can get the price of the contingent claim. Take Huaxia 50ETF(stock code:510050) option in China as the example which first appears to the market on 9th February, 2015. The price of this kind of options are computed under Black-Scholes model, the risk-neutral valuation and risk-exogenous valuation in this paper. Some parameters are given as follows:

the rate of exogenous risks  $\eta = 0.3\%$ ;

the expectation of jump size  $\theta = -0.5$ ;

the variance of jump size  $\delta = 1$ ;

the closing price of 50ETF on 6th February, 2015 is 2.291 yuan;

the strike price of the options are 2.20 yuan, 2.25 yuan, 2.30 yuan, 2.35 yuan and 2.40 yuan;

the delivery time of the options is June, so the expire time is  $\frac{4}{12}$ ;

the risk-free rate  $r = 4.92\%$  by choosing the three-year treasury bonds interest rates of China;

the volatility of 50ETF  $\sigma = 0.4097$ .

Then the prices of call and put options are obtained by MATLAB as Table 1 and Table 2.

Table 1: Price of call options

| strike price | actual price | BS formula | error  | equation(5.1) | error  | equation(5.2) | error  |
|--------------|--------------|------------|--------|---------------|--------|---------------|--------|
| 2.20         | 0.2815       | 0.2791     | 0.0024 | 0.2811        | 0.0007 | 0.2822        | 0.0004 |
| 2.25         | 0.2555       | 0.2531     | 0.0024 | 0.2552        | 0.0008 | 0.2563        | 0.0003 |
| 2.30         | 0.2314       | 0.2289     | 0.0025 | 0.2311        | 0.0007 | 0.2321        | 0.0003 |
| 2.35         | 0.2091       | 0.2064     | 0.0027 | 0.2087        | 0.0007 | 0.2098        | 0.0004 |
| 2.40         | 0.1885       | 0.1858     | 0.0027 | 0.1881        | 0.0006 | 0.1891        | 0.0003 |

Table 2: Price of put options

| strike price | actual price | BS formula | error  | equation(5.1) | error  | equation(5.2) | error  |
|--------------|--------------|------------|--------|---------------|--------|---------------|--------|
| 2.20         | 0.1560       | 0.1523     | 0.0037 | 0.1553        | 0.0007 | 0.1564        | 0.0004 |
| 2.25         | 0.1793       | 0.1755     | 0.0038 | 0.1787        | 0.0006 | 0.1798        | 0.0005 |
| 2.30         | 0.2044       | 0.2004     | 0.0040 | 0.2037        | 0.0007 | 0.2049        | 0.0005 |
| 2.35         | 0.2313       | 0.2272     | 0.0041 | 0.2305        | 0.0008 | 0.2318        | 0.0005 |
| 2.40         | 0.2599       | 0.2557     | 0.0042 | 0.2590        | 0.0009 | 0.2605        | 0.0004 |

The errors between actual price and valued price via every model are also showed to help us make the conclusion. Compared with classic BS model without jump which has an average error 0.0025 for call options and 0.0040 for put options, the jump-diffusion model can better characterize the option price whatever (5.1) and (5.2). Based on the jump-diffusion model, we compute the call and put option prices under the risk-neutral valuation principle as equation (5.1) and the risk-exogenous valuation principle as equation (5.2). The results show that the risk-neutral valuation has the same average errors 0.0007 for call and put options. While the

risk-exogenous valuation is better than the former two methods which has very small errors with the actual option price in the market, the average errors are 0.0003 and 0.0005 respectively.

**Acknowledgements** The authors are grateful to all the comments from editors and reviewers. This work was supported by National Nature Science Foundation of China(No.11871275; No.11371194) and by a Grant-in-Aid for Scientific Research from Nanjing University of Science and Technology(Grant Nos: KN11008,2011YBXM120) and

**Author contributions** All authors contributed equally and significantly this paper. All authors read and approved the final manuscript.

**Conflicts of interest** The authors declare no conflict of interest.

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