Engineering and Technology Journal e-ISSN: 2456-3358

Volume 07 Issue 07 July-2022, Page No.-1419-1421

DOI: 10.47191/etj/v7i8.04, I.F. – 6.484

Comparison of Modified D-H Notation with Standard D-H for and all of Direct Kinematics of Industrial Robotic manipulators

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SUMMARY: The accelerated growth of technology has allowed the development of more complex electromechanical structures and specific to the industry demand. Robots have emerged to meet this demand. The main objective of this work is to present a study of direct kinematics with the purpose of analyzing the behavior of the position and orientation of the industrial robotic manipulator in cartesian space in relation to a coordinate system. The methodology used in this exploratory scientific research will be developed, based on experimental tests, bibliographic references and case study applied at the Advanced Robotics Institute (IAR). The robot under study is the YASKAWA-MOTOMAN-GP7, from the IAR Robotics laboratory. The work contributes to an analysis and proof of Denavit-Hartenberg's notation. The results are the determination of the equations of direct kinematics of the robot, matrix equations developed in the MATLAB© software implemented capable of proving the mathematical model developed, which applies in all industrial robots.

KEYWORDS: Denavit-Hartenberg notation; Teaching industrial robotics; Direct Kinematics; Industrial Robotics.

I. INTRODUCTION

The technological concept of industries directs them to a new paradigm of production, launder productivity, increasing profitability, reducing costs and quality of processes. The history of industrial automation is characterized by periods of rapid technological change; the result of this has been the development of automatons capable of performing the most complex tasks with the highest level of sophistication and precision possible. It was precisely in this context that industrial robotic manipulators emerged [1].

The Denavit-Hartenberg (D-H) *notation* describes a serial link of the mechanism that is a fundamental tool for the study of kinematics. According to this definition and description of a manipulator, we can make use of technical algorithms to find kinematic, Jacobian, dynamic, motion planning and simulation solutions, for example [1], [2].

Vector algebra and matrix algebra are used to develop and describe the location of robotic arm links in relation to fixed coordinate systems. Since the links of a robot must have rotation and/or translation with respect to a reference coordinate system. The problem of direct kinematics is reduced to finding the homogeneous transformation matrix that relates the coordinate system aggregated to the body to a reference coordinate system [3]. Aiming at the study and the different practical applications, in the projects of manipulative robots two lines of research can be addressed, kinematics and dynamics. Kinematics deals with movement without considering the forces that cause this movement, the study of forces is part of the dynamics.

In the literature, the kinematics of manipulator robots are approached through two models, direct and inverse kinematics. Direct kinematics consists of finding the

Orientation and position of the terminal effector from the vector of angles of the joints and the geometric parameters of

The model. In [4], the authors proposed a matrix method for matrix modeling and solution of problems of mechanisms that need to be positioned and oriented in the Cartesian space.

Kinematics inverse is considered an effective technique of control of a robotic arm, consisting of finding the vector of angles of the joints, from the orientation and position of the terminal effector. It presents great challenges due to the nonlinearity of equations and multiple solutions for manipulators with many degrees of freedom [5], [6].

II. LITERATURE REVIEW

2.1 Direct Kinematics

Industrial robots generally have six degrees of freedom. The location of its final effector is specified by adequately controlling its joint variables, consequently the values of the set of joint variables of a robot, determine the positioning of its terminal element in the system of work coordinates [1]. In general, the initial three degrees of freedom of a robot has the role of positioning the final effector, and the other three are responsible for guiding it. To solve the problem of direct kinematics, there are several methods, the scope of this work includes *the Denavit-Hartenberg notation*.

2.1.1 DENAVIT-HARTENBERG RATING (D-H)

The Denavit-Hartenberg Notation is based on the fact that to determine the relative position of straight duas in space, only two parameters are required. The first parameter is the distance measured along the common normal between the two lines and the second is the angle of rotation around the common normal, which one of the lines must rotate so that it is parallel to the other. It is observed that the common normal between two lines in space is defined by a third line that intercepts the first two lines, with angles of 90°. In addition, the distance measured between the two lines, along the common normal, is the shortest distance between them. Figure 1 shows two lines in space and the two parameters needed to describe their relative position.

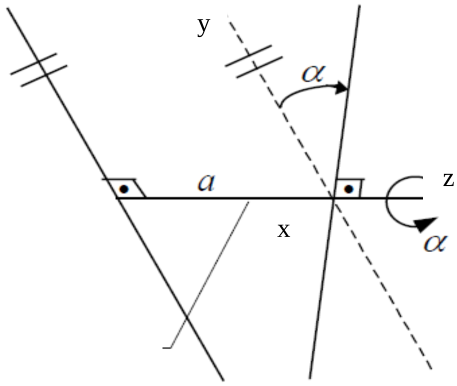

Figure 1. Relative position of two lines in the space Adapted from [6].

Denavit-Hartenberg proposed a systematic notation to assign an orthonormal coordinate system with the "right hand rule", one for each link in an open kinematic chain of links. Since these coordinate systems fixed to the link are assigned, transformations between adjacent coordinate systems can be represented by a homogeneous coordinate transformation matrix [6].

According to [8], in the original D-H representation, the ssocia the axis of the join to the z axis and each matrix is represented by the product of four basic transformations involving rotations and translations as we can observe in equation (1).

$$ {}^{k-1}T_k = R_{z,\theta} \cdot Transl_{z,d} \cdot Transl_{x,a} \cdot R_{x,\alpha} \qquad (1) $$

Figure 2 shows the D-H parameters with a graphical representation [1].

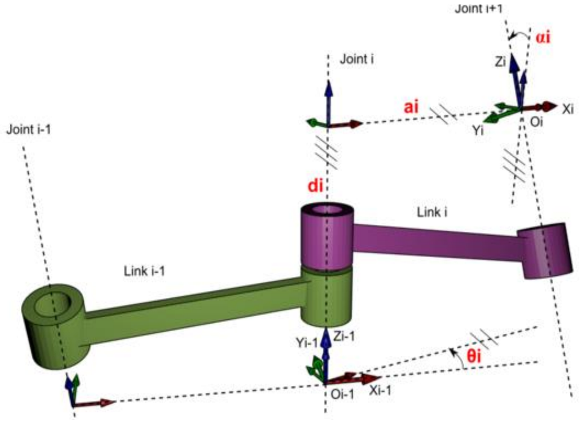

Figure 2. Representation of D-H parameters [3].

$$ {}^{i-1}T_i = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{array}{cccc} \text{Yaw} & \text{Pitch} & \text{Roll} & \\ \end{array} \begin{bmatrix} n & s & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Figure 3. Transformation matrix [12].

Figure 4 shows the detail of the movement of the fist representing with the initials N (yaw), S (pitch) and A (Roll).

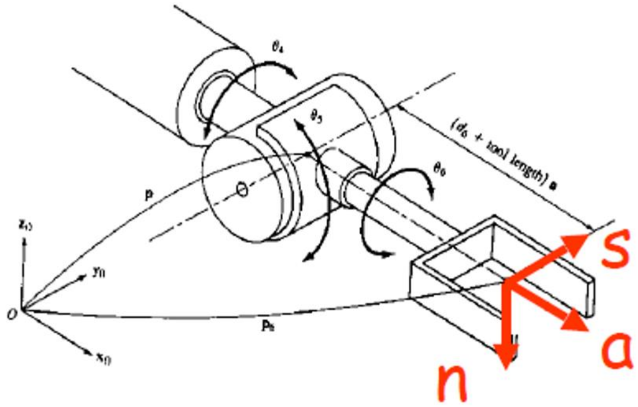

Figure 4. Representation of the acronym N, S, A of the wrist [12].

Table 1 shows that you can describe any homogeneous transformation matrix from four basic transformations.

Table 1.Basic Transformations of *Denavit-Hartenberg* [5].

Symbol	Meaning
$R_{z,\theta}$	Rotation around the Z axis by an angle θ
$T_{z,d}$	Translation along the Z axis by a distance d
$T_{x,a}$	Translation along the X-axis by a distance to
$R_{x,\alpha}$	Rotation around the X axis by an angle α

The calculations corresponding to a generic homogeneous transformation and each of the basic transformations are presented in equations 1 to 5.

$$H_{i-1}^{i} = R_{z,\theta_i} \cdot T_{z,d_i} \cdot T_{x,a_i} \cdot R_{x,\alpha_i} \tag{1}$$

$$R_{z,\theta_i} = \begin{bmatrix} cos\theta_i & -sen\theta_i & 0 & 0 \\ -sen\theta_i & cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$T_{z,d_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$T_{x,a_i} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$R_{x,\alpha_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha_i & -sen\alpha_i & 0 \\ 0 & sen\alpha_i & cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

To obtain the homogeneous transformation matrix (equation 6) that goes from the base to the end of the final effector, we have to multiply all the transformation matrices obtained by the D-H algorithm, that is, the resulting matrix can be considered as the solution of the problem of direct kinematics according to equation (7).

$$^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{6}$$

$$^{0}_{6}T = {}^{0}_{1}T.{}^{1}_{2}T.{}^{2}_{3}T.{}^{3}_{4}T\,.{}^{4}_{5}T\,.{}^{5}_{6}T \tag{7}$$

In the case of composite rotations around two or more axes the rotation component (orientation) of the transformation matrix will be more complex than that of the simple rotations indicated above. The homogeneous geometric transformation matrix has, in addition to rotations and translation plus four terms (the last line). Figure 5 shows the components of the 3D transformation matrix.

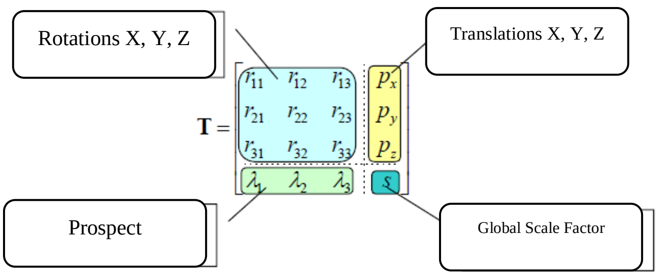

Figure 5. Components of the 3D transformation matrix [9].

2.1.1 Modified *Denavit-Hartenberg* Rating (DHM)

This idea is very important: one degree of freedom of rotation and one of position is lost. It has already been established that a convenient place to place the coordinate system is at the beginning or end of the segment (in relation to the robot's immovable base: on the axis of the distal or proximal joint). The original Denavit-Hartenberg notation uses the distal joint. However, this brings some drawbacks: the inversion of the model becomes more difficult [1]. Robot calibration tends to lose quality and calibration algorithms lose efficiency [10]. Therefore, the approach to be used is to locate the coordinate system in the joint closest to the robot's base (farther from the effector). This "new" notation is called DHM to differentiate it from the original in which the letter "M" comes from the word "modified". Note that the coordinate system moves with the segment considered and the previous segment is prismatic or rotary.

Figure 6 shows the D-H parameters with a graphical representation [1].

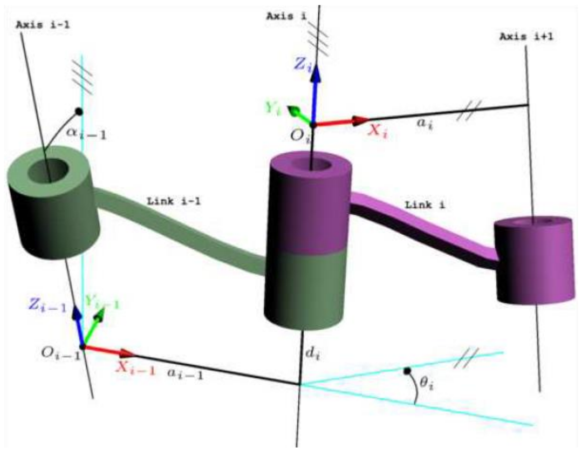

Figure 6. Representation of DHM parameters [1].

According to [1] the adopted matrix uses modified DH parameters. The difference between the classic D-H parameters and the DHM parameters are the coordinate system link locations to the links and the orders of the performed transformations. Another difference is that, according to the modified convection, the transformation matrix is provided in the following order of operations, as presented in equations (8) and (9).

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n)\cdot\mathrm{Rot}_{z_{n-1}}(\theta_n)\cdot\mathrm{Trans}_{x_n}(r_n)\cdot\mathrm{Rot}_{x_n}(\alpha_n) \tag{8}$$

$$^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & a_{n-1} \\ \sin\theta_n\cos\alpha_{n-1} & \cos\theta_n\cos\alpha_{n-1} & -\sin\alpha_{n-1} & -d_n\sin\alpha_{n-1} \\ \sin\theta_n\sin\alpha_{n-1} & \cos\theta_n\sin\alpha_{n-1} & \cos\alpha_{n-1} & d_n\cos\alpha_{n-1} \\ 0 & 0 & 0 & 1 \end{array}\right] \tag{9}$$

III.METHODOLOGICAL PROCEDURES

The methodology used in this exploratory scientific research will be developed, based on experimental tests, bibliographic references and case study applied at the Advanced Robotics Institute (IAR). The robot under study is the YASKAWA-MOTOMAN-GP7, from the IAR Robotics laboratory. This model was chosen because it is articulated type that is widely used in industries in general. The articulated type manipulator has its configuration similar to that of a human arm, due to its set of 6 rotational joints, which can be separated into two groups that make up the wrist and those corresponding to the movement of the arm. Figure 7 shows the squematic design of the robot under study.

Figure 7. Squematic design of the robot under Study [11].

The articulated type of manipulator has its configuration similar to that of a human arm, due to its set of 6 rotational joints, which can be separated into two groups that make up the wrist and those corresponding to the movement of the arm. Figure 8 shows the technical specifications of the robot of the manufacturer YASKAWA-MOTOMAN-GP7, with the dimensions of its structure in millimeters.

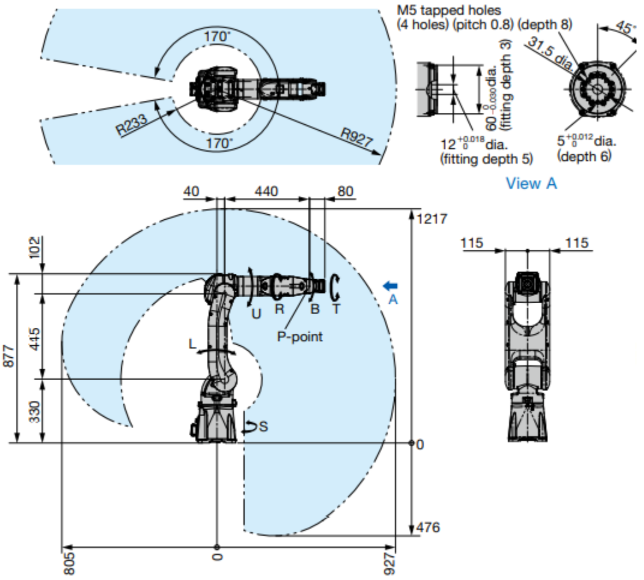

Figure 8. Technical specifications of the robot YASKAWA-MOTOMAN-GP7 [11].

One of the features that differs from this standard model of yaskawa-MOTOMAN-GP7 robot is the misalignment of the base center with the center of the manipulator end. This misalignment can be seen in Figure 4, which clearly shows that the link on which the handle is located 40 mm to the right with respect to the center of the robot base. To identify the respective joints of the Robot YASKAWA-MOTOMAN-GP7, figure 9 shows the joints j1, j2, j3, j4, j5 and j6 in a squematic way.

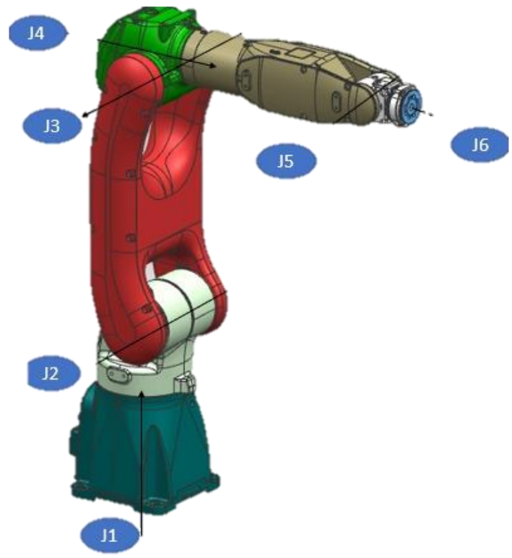

Figure 9. Squematic design of the YASKAWA-MOTOMAN-GP7 robot with the identification of their respective joints.

The approach adopted for the determination of direct kinematics of this robot is part of the preliminary analysis of possible movements and recognition of the types of links and joint components of the system. Then, based on the information acquired, coordinate systems are adopted for the axes to be studied with a view to determining *the Denavit-Hartenberg parameters of* the robotic arm under study. This is followed by the formation of the transformation matrices for each joint and the consequent composition of the robot's transformation matrix. After determining cartesian coordinate systems in each robot link, it is necessary to calculate the transformation matrices that represent the linear relative displacements between the links. The equation of direct kinematics has the direct kinematic problem that provides the final position of the robot effector. Once the transformation matrix of the reference system for the base (fixed) system is calculated, the direct kinematics relationships can be obtained. Once the values of a1 and a2 are known and the angles are given, the final position of the effector (Px, Py, Pz) is found. Through the equation of inverse kinematics, which is the inverse process of direct kinematics, we obtain the angles of the joints between the links from the final position of the effector.

3.1 Direct Position Kinematics

For the calculation of direct position kinematics, the literal table is mounted with the D-H parameters. The D-H parameters require a squeamatic model of the robot, as well as its characteristics, reference system and joining variables according to the convention *and Denavit-Hartenberg*. Figure 10 shows the representation of the "right hand rule" represented in the coordinates, developed for this case study.

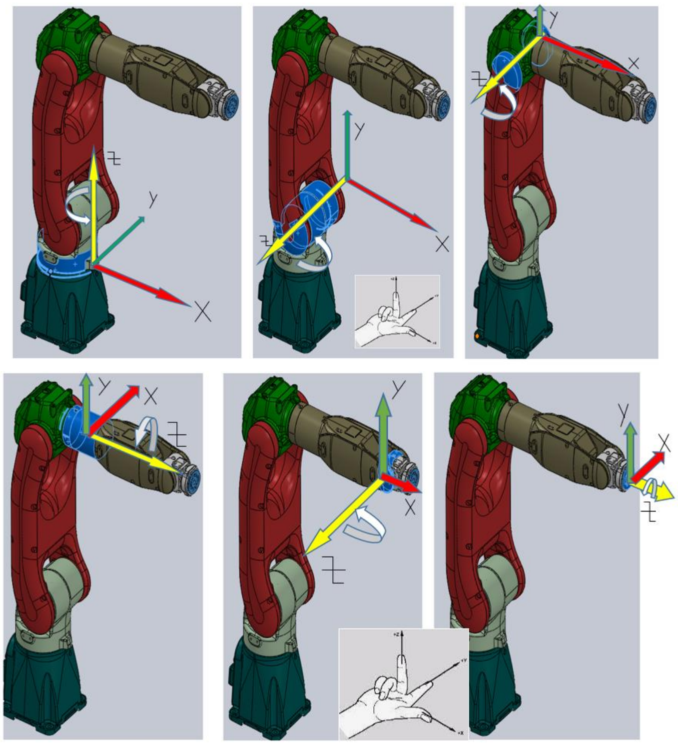

Figure 10. Graphical representation of frames, rotation in relation to coordinates (X, Y and Z).

From the squematic model, the *Denavit-Hartenberg table is defined*, according to table 2. The units used are millimeters for length and radians for angles.

Table 2. **Table Denavit-Hartenberg.**

Elo i	Axis	Joint	θ_i	α_i	a_i	d_i	Belt
1	S	0 → 1	-90	0	40	33	± 170
2	L	1 → 2	0	0	0	445	+145 - 65
3	U	2 → 3	0	0	0	102	+190 - 70
4	R	3 → 4	-90	90	0	440	± 190
5	B	4 → 5	-90	90	0	80	± 135
6	T	5 → 6	-90	-90	0	0	± 360

3.2 MATLAB©

MATLAB©, short *for MATrix LABoratory, is a simple* and direct language software used for mathematical calculations, which has high computational performance and a wide library of predefined mathematical functions. These features allow programming problems to be solved more simply than in other computational languages [12].

The *software* used to mathematically demonstrate and prove the *Denavit-Hartenberg* algorithm was developed in MATLAB©, R2021a by *mathworks*, *software often* used by researchers to perform calculations and systems in general [12].

The first step was todefine the *Denavit-Hartenberg parameters* for each joint andlink, replacing the actual values of angular relationship between one joint to another and the translation (in milímeters) between joints of the Robot YASKAWA-MOTOMAN-GP7. The values were obtained from the *manufacturer's* datasheet [11]. Then, by multiplyingthese matrices we obtain the homogeneous transformation matrix, which provides the mapping of the coordinates from the base to the end of the tool. In MATLAB©, each 'L' link is defined from 1 to 6 and then the YASKAWA-MOTOMAN-GP7 object that will represent the robot with the connection of its links in series, as we can see in Figure 11.

```
%Parâmetros de Denavit-Hartenberg descrevendo os tamanhos e relações
%angulares entre os eixos
L1 = link([pi/2   -0.075   0     0      0  ]);
L2 = link([0       0.3     0     0      0  ]);
L3 = link([pi/2    0.075   0     0      0  ]);
L4 = link([-pi/2   0       0    -0.32   0  ]);
L5 = link([pi/2    0       0     0      0  ]);
L6 = link([0       0       0    -0.08   0  ]);

Fanuc=robot({L1 L2 L3 L4 L5 L6});
Fanuc.name='Fanuc'
```

Figure 11. D-H parameters describing the angular sizes and relationships between the axes.

3.3 Validity

For validation, a test of the calculated code no"TP" *teach pendant of the* ROBOT FANUC model LR Mate 200iD will be performed, as shown in Figure 12.

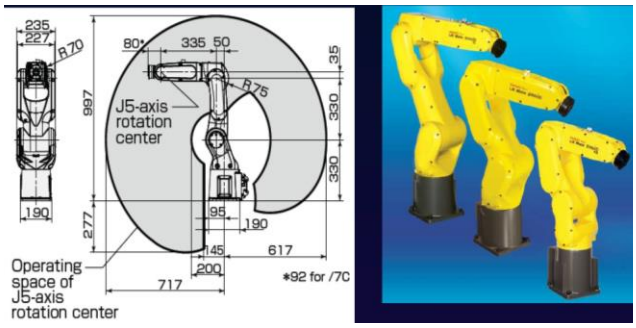

Figure 12. Technical specifications of the robot FANUC model LR Mate 200iD.

IV. RESULTS AND DISCUSSION

The methodology used to obtain the results regarding direct kinematics and inverse kinematics pointed to the initial analysis of the Structure of the YASKAWA-MOTOMAN-GP7 Robot and to the relationship of the Denavit-Hartenberg parameters for each axis of the robot. Then, these values were related through transformation matrices for the same axes. Finally, these matrices were summed up, giving rise to a matrix of final transformation of the robot, whose positioning at point T refers to the sum of the others (six links).

The Representation of D-H results in obtaining a homogeneous transformation matrix 4x4, representing that transforms the coordinates of the system from the i link to the i-1 system of the anterior link. With this, you can express the coordinate transformation from system "i" to system "i-1". The kinematic modeling ready to be applied to the controller is then obtained for experimental tests and system control simulations. By calculating direct kinematics it is possible to determine the x and y coordinates, once the coordinates of the joints are known. The resulting matrix can be considered as the solution of the problem of direct kinematics as shown in Figure 13, that is, the product of all transformation matrices obtained by D-H results in the solution of direct kinematics.

Figure 10 presents the homogeneous transformation matrix obtained by D-H, developed literally.

$$
{}^{0}T_{1}=\begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{1}T_{2}=\begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2.c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2.s\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{2}T_{3}=\begin{bmatrix} c\theta_3 & 0 & s\theta_3 & a_3.c\theta_3 \\ s\theta_3 & 0 & -c\theta_3 & a_3.s\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}T_{4}=\begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{4}T_{5}=\begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{5}T_{6}=\begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Figure 13.Result of transformation matrices obtained by D-H.

Figure 14 shows the homogeneous transformation matrix obtained by D-H, developed numerically. We highlight in yellow the value of the result of the resultof the homogeneous transformation matrix, obtained from the original D-H.

$$
{}^{6}_{0}T= {}^{0}_{1}T\begin{matrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 330 \\ 0 & 0 & 0 & 1 \end{matrix}
\;{}^{1}_{2}T\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 445 \\ 0 & 0 & 0 & 1 \end{matrix}
\;{}^{2}_{3}T\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 102 \\ 0 & 0 & 0 & 1 \end{matrix}
\;{}^{3}_{4}T\begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 440 \\ 0 & 0 & 0 & 1 \end{matrix}
\;{}^{4}_{5}T\begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 90 \\ 0 & 1 & 0 & 80 \\ 0 & 0 & 0 & 1 \end{matrix}
$$

$$
{}^{5}_{6}T\begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 90 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} /
$$

****

Resultado $\quad {}^{6}_{0}T=\begin{matrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 170 \\ 0 & 0 & 0 & 775 \\ 0 & 0 & 0 & 1 \end{matrix}$

****

Figure 14. Result of transformation matrices obtained by D-H.

Figure 15 shows the product of the multiplication of homogeneous transformation matrices, obtained by the D-H "DH", developed literally, that is, the resulting product of the matrices, in this case the solution for the direct kinematics of the robot YASKAWA-MOTOMAN-GP7.

$$
T_1^0=\begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};
\quad
T_4^3=\begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix};
\quad
T_6^0 = T_1^0.T_2^1.T_3^2.T_4^3.T_5^4.T_6^5
$$

$$
T_2^1=\begin{bmatrix} c_2 & -s_2 & 0 & a_2.c_2 \\ s_2 & c_2 & 0 & a_2.s_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix};
\quad
T_5^4=\begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};
\quad
T_6^0=\begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
T_3^2=\begin{bmatrix} c_3 & 0 & s_3 & a_3.c_3 \\ s_3 & 0 & -c_3 & a_3.s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};
\quad
T_6^5=\begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Figure 15. Result of the transformation matrices obtained by the D-H "DH", obtained by the literal form.

Figure 16 shows the product of the multiplication of homogeneous transformation matrices, obtained by the D-H "DH", developed numerically, that is, the resulting product of the matrices, in this case the solution for the direct kinematics of the robot YASKAWA-MOTOMAN-GP7.

```
>> TR=T1*T2*T3*T4*T5*T6

TR =

    0.0000     -1.0000     -0.0000     40.0000
   -1.0000     -0.0000     -0.0000    -400.0000
    0.0000      0.0000     -1.0000     330.0000
    0           0           0           1.0000
```

Figure 16. Result of the transformation matrices obtained by the D-H "DH", obtained numerically.

Figure 17 shows the homogeneous transformation matrix obtained by D-H modifies "DHM", developed literally.

$$
{}^{n-1}T_n=\begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & a_{n-1} \\ \sin\theta_n\cos\alpha_{n-1} & \cos\theta_n\cos\alpha_{n-1} & -\sin\alpha_{n-1} & -d_n\sin\alpha_{n-1} \\ \sin\theta_n\sin\alpha_{n-1} & \cos\theta_n\sin\alpha_{n-1} & \cos\alpha_{n-1} & d_n\cos\alpha_{n-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Figure 17. Result of transformation matrices obtained by the modified D-H "DHM".

Figure 18 shows the homogeneous transformation matrix obtained by the modified D-H "DHM", developed numerically.

```
TRM =

    0.0000   -1.0000   -0.0000   40.0000
   -1.0000   -0.0000   -0.0000   45.0000
    0.0000    0.0000   -1.0000  250.0000
         0         0         0    1.0000
```

Figure 18. Result of transformation matrices obtained by the modified D-H "DHM".

Figure 19 shows the product of the multiplication of homogeneous transformation matrices, obtained by the modified D-H "DHM", developed literally, that is, the resulting product of the matrices, in this case the solution for the direct kinematics of the robot YASKAWA-MOTOMAN-GP7.

```
>> TRL = T1L*T2L*T3L*T4L*T5L*T6L

TRL =

[cos(theta6)*(sin(theta5)*(sin(alpha4)*(cos(alpha3)*(cos(alpha2)*sin(alpha1)*sin(theta1) + sin(alpha2)*cos(theta1)*sin(theta2) + cos(alpha1)*sin(alpha2)*cos(theta2)*
[sin(theta6)*(sin(alpha5)*(cos(alpha4)*(sin(alpha3)*sin(theta3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2)) - cos(alpha3)*(cos(alpha2)*sin(alpha
[
[

>> TR = T1*T2*T3*T4*T5*T6

TR =

    0.0000   -1.0000   -0.0000   40.0000
   -1.0000   -0.0000   -0.0000 -400.0000
    0.0000    0.0000   -1.0000  330.0000
         0         0         0    1.0000
```

Figure 19. Result of the transformation matrices obtained by the modified D-H "DHM", obtained by the literal form.

Figure 20 shows the product of the multiplication of homogeneous transformation matrices, obtained by the modified D-H "DHM", developed numerically, that is, the resulting product of the matrices, in this case the solution for the direct kinematics of the robot YASKAWA-MOTOMAN-GP7.

```
>> TR = T1*T2*T3*T4*T5*T6

TR =

    0.0000   -1.0000   -0.0000   40.0000
   -1.0000   -0.0000   -0.0000 -400.0000
    0.0000    0.0000   -1.0000  330.0000
         0         0         0    1.0000
```

Figure 20. Result of the transformation matrices obtained by the modified D-H "DHM", obtained numerically.

Figure 21 shows the coordinates of the axes in X=P_x, Y=$P_{y,}$ and Z=P_z of the Robot YASKAWA-MOTOMAN-GP7.

$$ {}^{i-1}T_i = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

40.0000

-400.0000

330.0000

1.0000

Figure 21.Result of coordinates X, Y and Z.

The purpose of direct kinematics of robots is to obtain a description of the position and orientation of the tool in função of the angles (in the case of rotating joints) or length (in the case of prismatic joints) of robot joint that are called "joint variables". It is important that the robotic manipulator is always *in its home position* position (with links to 0° or 90° of each other) to facilitate the convention of d-h parameters and the possibility of creating a reliable algorithm as shown in Figure 22.

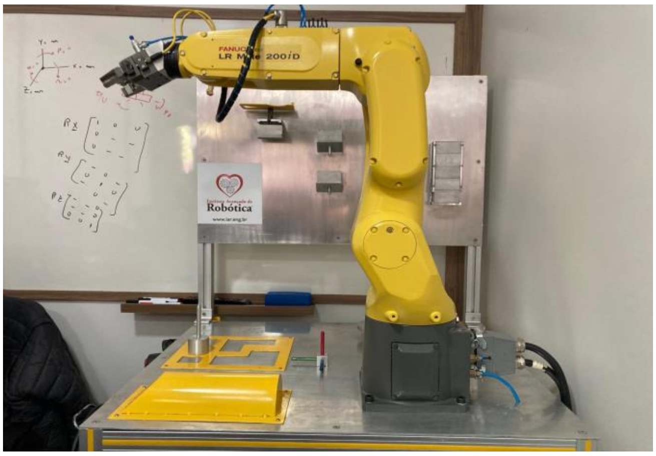

Figure 22. Result of positioning the robot in *home position*.

Figure 23 shows the ROBOT FANUC model LR Mate 200iD not being in the home *position position*. There is great difficulty in establishing the extraction of frames for comparative analysis between the modified D-H notation comparison model with standard D-H for the study of direct kinematics of industrial robotic manipulators.

It is extremely important that to extract the frames the robot is in the home position position, for better result and comparison of calculations in relation to the spatial position of the robot physically.

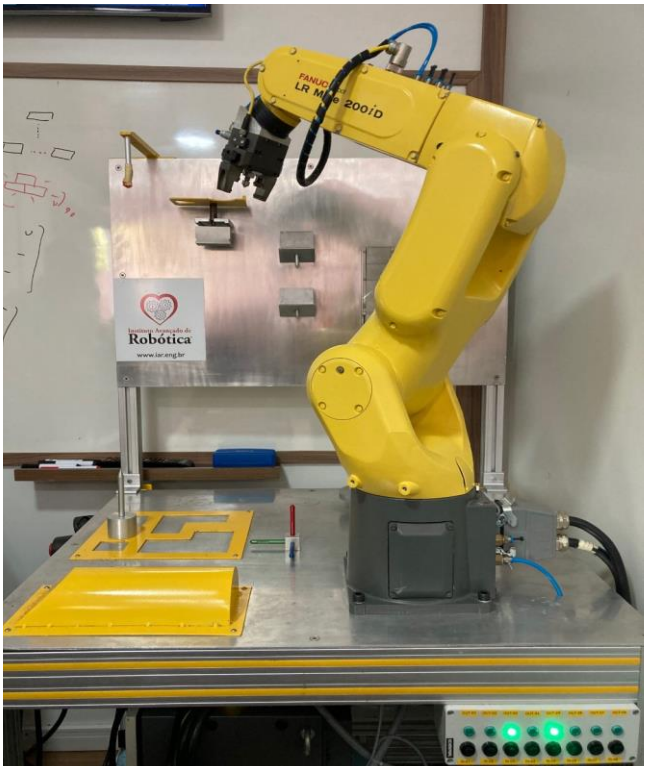

Figure 23. **Result of positioning the robot in home position.**

According to [1] the most systematic methodology and structure determines that the robot joints must be numbered in ascending order, starting at the base of the manipulator, and ending with the last link that in this case was the claw.
To validate the information contained in the "TP" figure 24 shows the actual data validated in the ROBOT FANUC model LR Mate 200iD.

Figure 24. **Result of positioning the robot in the "TP"** ***teach pendant.***

The difference between conventional D-H parameters and modified D-H "DHM" are the locations of the coordinate systems and the order of the transformations.

V. CONCLUSION

The results show the determination of the equations of the direct kinematics of the robot YASKAWA-MOTOMAN-GP7, matrix equations of the *classical Denavit-Hartenberg* "D-H" notation were implemented and compared with the modified *Denavit-Hartenberg notation* "DHM". In addition, with them multiplication of the matrices we obtain the homogeneous transformation matrix, which provides the mapping of coordinates from the base to the extremify the tool.
With the help of MATLAB© software we were able to calculate homogeneous transformation matrices, which applies in all industrial robots. It is extremely important that to extract the frames ofthe robot with maximum precision it is necessary that the robot is in the home position position, for better result and comparison of calculations in relation to the spatial position of the robot physically. This paper presents a contribution to the Advanced Institute of Robotics, as they were skills acquired in the curricular unit of the discipline of modeling robotic manipulators of the postgraduate course in Robotic Engineering.

THANKS

The authors thank the Advanced Institute of Robotics - IAR, which ceded its structure of robots and *software* to enable the realization of this work.

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