# Prediction of Movement of a Large Boulder in the Channel of the Mountain Stream 

Irma Inashvili ${ }^{1}$, Konstantine Bziava $^{2}$, Khatuna Soselia ${ }^{3}$<br>${ }^{1}$ Dr., Professor, Georgian Technical University, Faculty of Civil Engineering, M. Kostava str. 77, 0171, Tbilisi, Georgia, ORCID: 0000-0002-6202-4580.<br>${ }^{2}$ Dr., Associate Professor, Georgian Technical University, Faculty of Civil Engineering, M. Kostava str. 77, 0171, Tbilisi, Georgia, ORCID: 0000-0003-1237-5224<br>${ }^{3}$ Ph.D Student, Georgian Technical University, Faculty of Civil Engineering, M. Kostava str. 77, 0171, Tbilisi, Georgia


#### Abstract

As it is known, according to the type of formation, density and fractional composition, debris flow is divided into two dynamic categories - cohesive (turbulent) and adhesive (hyper-concentrated, structural). Cohesive debris flows, the contents of which are represented in water as a soil suspension, obey the general laws of hydromechanics. Adhesive debris flow is not characterized by turbulent interchange, which is caused by saturation of the flow with fine fractions and adhesive water. Since, in certain sections of the transit zone of the flow, there are such changes in morphometric characteristics that are often not subject to assessment; the analytical solution of hydraulic problems of debris flows does not lose its relevance and is the object of research of many researchers. According to the above mentioned the article deals with the problem of the rectilinear motion of a boulder in the streambed with a positive bottom slope under the influence of water flow and gravity of a boulder. We obtained the equation for calculation of the average velocity of the water flow in order to transport a large boulder, which is partially submerged in the stream.


KEY WORDS: water flow, average velocity, stream channel, debris flow, debris flow density, viscosity, channel slope.

## Introduction

Catastrophic debris flow, especially in mountainous and foothill areas, often cause destabilization and destruction of cities and other settlements, railways and highways, electrical and communication transmitters, canals, pipelines, agricultural land, recreational sites and different parts of infrastructure. Therefore, improvement of the methods applied for selection of reliable and sustainable debris flow control engineering structures and constructions remains an important issue for environmental protection and safety.

## MAIN PART

As it is known, the movement of a large boulder in the bed of a mountain watercourse usually occurs at a lower velocity than the movement of the transporting (water) stream itself. Currently, there is no clear method for determining the conditions of deceleration in the water flow. When solving the problem of interaction of incoherent (water-stone) streams with structures, the mentioned retard is of significant importance, since the force of impact on an obstacle also depends on the velocity of its movement. The problem of predicting the length of the path of movement of a large boulder in the channel of the watercourse with a decrease in the slope of the bottom of the watercourse has not been solved [1, 2, 3, 4, 5, 6].

Based on the above, the purpose of this work is to solve the problem of rectilinear motion of a boulder in the channel of a watercourse with a positive slope of the bottom under the influence of the water flow and gravity of the boulder. The obtained equations should be applied in the design of engineering debris flow control hydraulic structures.

Below is the solution to the problem of the rectilinear motion of a boulder in a channel with a positive slope of the bottom under the influence of water flow and gravity.

To simplify the problem, assume that a boulder has a spherical shape, immersed in a water flow (Fig. 1).


Fig. 1. Scheme of the movement of a large spherical rock fragment in the stream channel
Designations: $V w$ - water velocity ( $\mathrm{m} / \mathrm{s}$ ); $F_{1}$ - force directed against the flow ( $N$ ); $d$ - diameter (m); $G$ - gravity (N); $G y$ projection of gravity on the $y$ axis; $G x$ - projection of gravity on the $x$ axis; $\theta$ - angle of inclination of the channels bed $\left({ }^{\circ}\right)$; $i$ - hydraulic slope ( $\mathrm{m} / \mathrm{m}$ ); $N$ - projection of the force caused by the flow around the surface.

The movement of the boulder is provided by the frontal action of the forces of the water flow and the gravity itself. The force of its friction against the bottom of the channel resists movement. With the designation $G_{y} / N=K$, the force directed against the flow of the stream will be determined by the equation:
$F_{1}=f\left(G_{y}+N\right)=f N(1+K)$
where $\quad N=\omega_{m} \rho_{w} \frac{V^{2}}{2}$ is the projection of the
force (pressing) caused by the flow around the boulder surface;
$\omega_{m}$ - midship surface of streamlined boulder surface area $\left(\mathrm{m}^{2}\right)$;
$\rho_{\mathrm{w}}$ - density of water ${ }^{1}\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$;
$G_{y}=G \cos \theta-$ the projection of the gravity
$(\mathrm{G})$ on the axis y ;
$\theta$ - the angle of inclination of the watercourse bottom in relation to the horizontal plane $\left({ }^{\circ}\right)$;
$f$ - coefficient of sliding ${ }^{2}$ friction of the boulder against the bed of the channel; designation $K=G_{y} / N$.

The sum of the projection of forces (on the abscissa axis $x$ ) acting on the boulder will be [7]:
$F_{x}=\frac{\pi d^{2}}{8}\left[K_{c}-f(1+K)\right] \rho_{w} V^{2}+\frac{\pi d^{3}}{6}\left(\rho_{o}-\rho_{w}\right) g i$

Where: $d$ is the boulder diameter (m);
$\rho_{o}$ - boulder density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$;
$V$ - relative flow velocity ( $\mathrm{m} / \mathrm{s}$ );
$K_{\mathrm{c}}$ - coefficient of hydraulic resistance;
$i$ - slope;
$g$ - acceleration of gravity $\left(\mathrm{m} / \mathrm{s}^{2}\right)$.
It is assumed that the water flow moves in a uniform mode. Using the Chézy formula, we will have:

$$
\begin{equation*}
I=i=\frac{V^{2}}{c^{2} R} \tag{3}
\end{equation*}
$$

where
$I$ is the hydraulic slope $(\mathrm{m} / \mathrm{m})$;
$R$ - hydraulic radius (m);
$C$ - Chezy's coefficient (velocity coefficient) ( $\mathrm{m}^{0,5} / \mathrm{s}$ ).

Taking into account (3), equation (2) becomes:
$F_{x}=\frac{\pi d^{2}}{8}\left[K_{c}-f(1+K)\right] \rho_{w} V^{2}+\frac{\pi d^{y}}{6}\left(\rho_{o}-\rho_{w}\right) \frac{V^{2}}{c^{2} R} g$
(4)

On the other hand, the projection of the force $F$ on the $x$ axis is:

$$
\begin{equation*}
F_{x}=m \frac{m^{2} x}{d t^{2}} \tag{5}
\end{equation*}
$$

where

$$
m=\frac{\pi d^{\mathrm{s}}}{6}\left(\rho_{\mathrm{o}}-\rho_{w}\right)-\text { mass of a }
$$

spherical boulder in water ( kg );
$t$ - time of movement in the stream (s).
Taking into account (5), instead of equation (4), we will have:
$\frac{d^{2} x}{d t^{2}}=\frac{d V_{z}}{d t}=\left\{\frac{3}{4} \frac{\left[K_{c}-f(1+K)\right] \rho_{w}}{d\left(\rho_{0}-\rho_{w}\right)}+\frac{g}{c^{2} R}\right\} V^{2}$

If we signify the relative velocity of the water through the equation $V=V_{0}-V_{\mathrm{w}}$, where $V_{\mathrm{o}}$ and $V_{\mathrm{w}}$ accordingly is the motion of movement of the boulder and water, then after simple transformations, instead of (6), taking into account $V_{\mathrm{w}}=$ const, we obtain that:
$-\frac{d V_{X}}{\left(V_{0}-V_{W}\right)^{2}}=E d t$
where $E$ is the surface area of contact and water $\left(\mathrm{m}^{2}\right)$ :
$E=\frac{3}{4} \frac{\left[K_{c}-f(1+K)\right] \rho_{w}}{d\left(\rho_{0}-\rho_{w}\right)}+\frac{g}{c^{2} R}=$ const
After integration, taking into account the boundary conditions (at $t=0$ and $V_{0}=0$, and the constant of integration $C_{1}=1 / V_{\mathrm{w}}$ ), we will get:
$V_{\mathrm{o}}=\frac{d x}{d t}=\frac{E V_{W}{ }^{2} t}{E V_{W} t+1}$
or
$\stackrel{\text { or }}{d x}=\frac{\Omega V_{W} t}{\Omega t+1} d t$
where $\Omega$ is the free cross-sectional area $\left(, \mathrm{m}^{2}\right)$ :
$\Omega=E V_{w} t=$ const
Integration of (10), taking into account the boundary conditions (at $t=0, V_{0}=0$, and the constant of integration $C_{2}=-1 / E$ ), provides:
$x=V_{w} t-\frac{\ln \left(E V_{W} t+1\right)}{E}$

Equation (12) allows to set the length of the rectilinear movement of a fixed boulder in the channel for a certain period of time, which makes it possible to estimate the average velocity of movement of a boulder in a watercourse:

$$
\begin{equation*}
V_{\mathrm{o}}=\frac{x}{t} \tag{13}
\end{equation*}
$$

It should be noted that the values of $E$ differ from each other not only depending on the shape of the boulder (e.g., spherical, cubic, elongated parallelepiped, or ellipsoid), but also on the orientation of the velocity vector $V_{W}$ of the progressive flow with respect to the boulder. These values are given in [8].

The shapes and sizes of the transported boulders can be different. Boulders, in which the shape differs from spherical, in the first approximation can also be characterized by an equivalent diameter, which is determined by the equation:
$d_{e q}=\sqrt{\frac{6 W}{\pi}}$
where $W$ is the volume of the boulder other than spherical $\left(\mathrm{m}^{3}\right)$.


Fig. 2. Scheme for calculating the movement by a water stream of a partially submerged large-sized rock fragment of any configuration

In the case when a boulder (of any shape) is partially submerged in the water flow (Fig. 2), the energy equation between the normal section 1-1 and section 2-2, which passes through the crest of a large boulder, similarly to [9,10], has view:
$H+\frac{W_{W}^{2}}{2 g}=H+\Delta H+\frac{V_{W}^{2}}{2 g}\left(\frac{\gamma_{W} W_{1}}{\gamma_{W} W_{1}+\gamma_{0} W_{2}}\right)^{2}+\frac{G_{b} \sin \theta}{\gamma_{a v} \Omega_{1}}-\frac{f_{c} \cos \theta}{\gamma_{a v} \Omega_{1}}$
where $\quad \Delta H$ is an excess of the height of the boulder over the free surface of the stream (m);
$H$ - normal water flow depth (m);
$\gamma_{w}$ - specific gravity of water $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$;
$\gamma_{o}$ - specific gravity of a boulder $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$;
$W_{1}$ - the volume of the part of the boulder immersed in the water flow $\left(\mathrm{m}^{3}\right)$;
$W_{2}$ - the volume of the part of the boulder located above the free surface of the water flow $\left(\mathrm{m}^{3}\right)$;
$W=W_{1}+W_{2}$ - the full volume of the boulder $\left(\mathrm{m}^{3}\right)$;
$V=V_{\mathrm{w}}+V_{\mathrm{b}}$ - the relative velocity of the water flow ( $\mathrm{m} / \mathrm{s}$ );
$V_{\mathrm{b}}$ - boulder movement velocity ( $\mathrm{m} / \mathrm{s}$ );
$\gamma_{a v}$ - relative specific gravity, partially submerged in a water stream $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$;
$\Omega_{1}$ - the area of contact with the bottom of
the watercourse, through which pressure is transmitted to the bottom of the channel $\left(\mathrm{m}^{2}\right)$;
$\boldsymbol{f}_{c}$ - coefficient of friction on the surface
of the watercourse;
$G_{b}$ - the weight of a boulder partially
submerged in a stream.
Weight of a boulder partially submerged in a stream:

$$
\begin{equation*}
G_{b}=\gamma_{w} W_{1}+\gamma_{o} W_{2} \tag{16}
\end{equation*}
$$

The relative specific gravity of a boulder partially submerged in a water flow is determined by the equation:

$$
\begin{equation*}
Y_{a v}=\frac{\gamma_{w} W_{1}+\gamma_{0} W_{2}}{W} \tag{17}
\end{equation*}
$$

Attention should be paid to the expression in parentheses on the right side of equation (15). It can be considered as a correction factor that takes into account, in the first approximation, the factor of the retarding the boulder from the water flow.

If we accept the designation:
$K_{1}=\frac{\gamma_{W} W_{1}}{\gamma_{W} W_{1}+\gamma_{K} W_{2}}$
for the weight coefficient of a boulder, partially submerged in the water flow, then instead of (15) we obtain:

$$
\frac{V_{W}^{2}}{2 g}\left(1-K_{1}^{2}\right)=\Delta H+\frac{c_{b}}{\gamma_{a v} \Omega_{1}}\left(f_{b} \cos \theta-\sin \theta\right)
$$

After simple conversion, expression (19) will take the form:
$V_{w}=\sqrt{\frac{2 g \Delta H+\frac{G_{b}}{\gamma_{a v} \Omega_{1}}\left(f_{b} \cos \theta-\sin \theta\right)}{\left(1-K_{1}{ }^{2}\right)}}$
Dependence (20) allows estimating the required value of the average water flow rate, which is needed for transporting a large boulder partially submerged in the flow.

Taking into account the change in the amount of movement along the path can be approximately estimated by the ratio:

$$
\begin{equation*}
K_{a v}=\frac{i_{1}}{i_{2}} \tag{21}
\end{equation*}
$$

Where: $i_{1}$ is an average slope of the channel of the watercourse transit zone;
$i_{2}$ - average slope of the channel bottom on the debris-flow cone.

To solve the next stage of moving a boulder, as a first approximation, we can use an equation:

$$
\begin{equation*}
\omega=\frac{\omega_{W} V_{W}^{z}}{K_{a v} V^{z}} \tag{22}
\end{equation*}
$$

We consider $K_{\text {av }}$ to be approximately constant in individual sections of the watercourse, substituting (22) into the dynamic equilibrium equation:

$$
\begin{equation*}
\frac{\gamma_{w} \omega d x}{g} \frac{d V}{d t}+\lambda X \frac{V^{2}}{2 g} d x=0 \tag{23}
\end{equation*}
$$

where $\quad \lambda$ is the coefficient of surface roughness of the debris-flow cone;
$\chi$ - wetted perimeter of the channel on the debris-flow cone;
let $\chi=b_{0}+2 x \tan \theta_{1}$, where the flow expands;
$b_{0}$ - width of the rectangular channel at the end of the transit zone (m);
$\theta_{l}$ - angle of free flow on the debris-flow cone.

Assuming $K_{\text {av }}$ to be approximately constant and substituting (22) into (23) taking into account the boundary conditions, we obtain:
$V_{a v}=V_{w} \sqrt{\frac{\omega_{w}}{\lambda K_{a v} b_{0} x+\lambda x^{2} \tan \theta_{1}+\omega_{w}}}$

Equation (25) makes it possible to determine the average velocity in the section located at a distance $x$ from the initial section.

## CONCLUSIONS

The resulting new dependencies make it possible to establish the length of the rectilinear movement of a fixed boulder in the channel for a certain period of time, which, in turn, makes it possible to estimate the average velocity of movement of a boulder in a watercourse and to judge the necessary value of the average velocity of the water flow, which is needed to transport a large boulder, partially submerged into the stream.

A new approach to solving engineering problems of the movement of debris flows and their application in the design of debris flow control structures will provide reliable protection of territories from the destructive impact of debris flows.

## REFERENCES

1. Takahashi, T. (2007). Debris Flow: Mechanics, Prediction and Countermeasures (Balkema: Proceedings and Monographs in Engineering, Water and Earth Sciences). Taylor and Francis.
2. Takahashi, T. (2006). Mechanisms of sediment runoff and countermeasures for sediment hazards, Kinmirai Sha.
3. Schuster, R.L. (2000). Outburst debris-flows from failure of natural dams, in G.F. Wieczorek and N.D. Naeser (eds.), Debris-flow Hazards Mitigation, Proc., 2nd Int'l. Conf. on Debris-Flow Hazards Mitigation, Taipeh, 16-18 August, pp. 29-42.
4. Harris, C., Vonder Muhll, D., Isaksen, K., Haeberli, W., Sollid, J. L., King, L., Holmlund, P., Dramis, F., Guglielmin, M. \& Palacios, D. (2003). Warming permafrost in European mountains. Global Planet Change, 39, pp. 215-225.
5. Coussot, P. (1997). Debris flow Rheology and Dynamics, IAHR Monograph Series, A.A. Balkema: Rotterdam.
6. Perov, W.F. (2012). Debris flow management. Faculty of Geography, Moscow State University. Moscow.
7. Natishvili, O.G., Tevzadze, V.I. (2002). The work of the water stream on the movement of the boulder and forecasting the ecological situation in the channels of the mountain water flow. Engineering ecology, 5, pp. 34-38.
8. Natishvili, O.G., Tevzadze, V.I. (2001). Movement of debris flows and their interaction with structures. Tbilisi.
9. Natishvili, O.G., Tevzadze, V.I. (2009). On the mechanism of moving a large-sized rocky fragment by a debris flow stream. Meteorology and Hydrology, 6, pp. 94-98.
10. Natishvili, O.G., Kruashvili, I.G., Inashvili, I.D. (2018). Applied problems of the dynamics of coherent debris flows. Nauchtekhlitizdat, Moscow.
