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Estimate Anti-symmetrical Divergence Modes of an Aircraft Wing Utilizing Aero-structure Analysis via Aerodynamic Lifting Line Theory

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ABSTRACT: This work illustrates the use of numerical matrix iteration to determine an aircraft wing's divergence speed using the aerodynamic span effects. The work starts with the construction of equilibrium equations in differential and integral forms. In this paper, integral formulas are used because they provide a convenient basis for numerical solutions of complex practical problems. Second, the straight-tapered wing has been divided into a number of Multhopp stations. Subsequently, the stations' torsional influence coefficient matrix has been calculated. Third, lifting line theory was used as a suitable choice of aerodynamic theory, and the governing equations were represented in matrix form. Finally, aerodynamic span effects are taken into consideration with induction effects according to Prandtl's lifting line theory to calculate the anti-symmetrical divergence speed from the lowest eigenvalue of the homogenous governing integral equation. To get the solution to converge, a matrix has been iterated via the MATLAB program.

KEYWORDS: Aero-elasticity, Anti-symmetrical divergence speed, Lifting line Theory, Matrix Iteration, Multhopp's -stations, Numerical solution, Span effects.

I. INTRODUCTION

Aero-elasticity is the study of how inertia, elastic, and aerodynamic forces interact in a flexible structure, as well as the phenomena that may arise [2]. The influence of inertial forces is introduced in dynamics. Elasticity describes the shape of an elastic body under a certain load. Classical aerodynamic procedures evaluate the forces acting on a body with a specific shape [3]. Aeroelastic phenomenon often fall into two categories as static and dynamic. The classic collar aeroelastic triangle [4], seen in Fig.1, briefly explains this phenomenon, as found in ref [1].

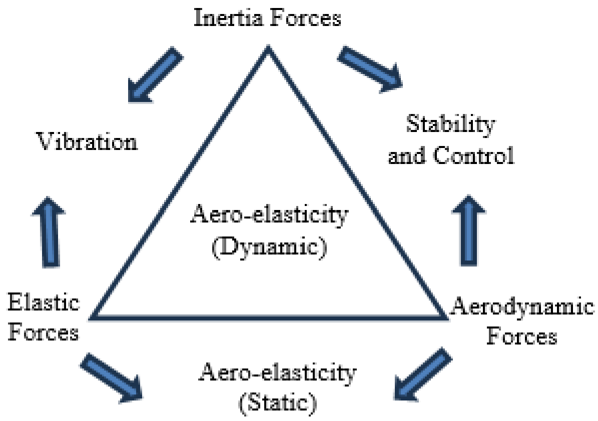

Fig. 1. The aeroelastic triangle of Collar [1].

II. DESCRIPTION OF THE WING STRUCTURE

The wing that is being studied is thought to be perfectly elastic. indicates that the wing structure will maintain its original shape after the external loads are removed. Experiments on airplane structures demonstrated that, within certain limitations, force and deflection are linearly connected. Elastic buckling in an aircraft wing structure's skin can result in a discontinuity in the force deflection diagram even in cases where the material that makes up the structure is under low stress [1]. Consequently, the elastic properties of the wing structure are reported in the range below the elastic buckling point.

III. MATHEMATICAL CONCEPTS

A. Understandable Influence Coefficients

The influence coefficients idea is used throughout this study, which accounts for wing structural deflections generated by various loads. This approach represents each point's total angular and linear deformation as the sum of the deformations at that point generated by individual forces and moments. This can be represented using the superposition concept, which is the basis for linear system analysis [6]. In this paper, the general situation is considered as an example as described in ref [1].

B. Utilizing influence coefficients to express strain energy

In order to apply the energy approach to an aeroelastic system, the strain energy formulas in relation to influence coefficients must be constructed to finally obtain the following relationship among the applied torsional moment and rate of twist:

$$\frac{d\theta(y)}{dy} = \theta' = \frac{T}{GJ} \qquad (1)$$

The proof of this equation is explained in more detail in ref [1].

C. Coefficients of torsional influence

Consider the cantilever wing in Fig.2, which is subjected to a unit torque force. At a distance η from the origin, a unit torque about the elastic axis is exerted, and the angular deflection resulting at y is denoted as $C^{\theta\theta}(y,\eta)$.

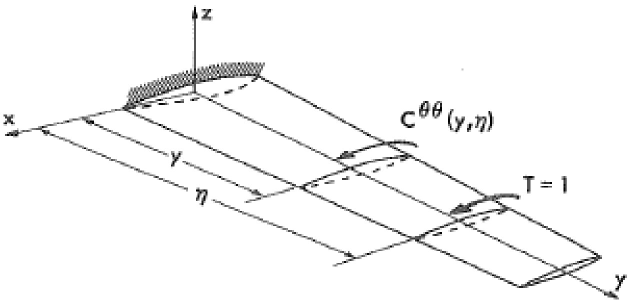

Fig. 2. Cantilever wing subjected to unit torque [7].

As stated in ref [1],

$$q(\lambda,s) = \vartheta(\lambda,s) \ldots.(0<\lambda<\eta)$$

$$\frac{\partial q}{\partial T} = \vartheta(\lambda,s)\ldots.(0<\lambda<y)$$

Thus, for $\eta > y$

$$C^{\theta\theta}(y,\eta) = \int_0^y \frac{d\lambda}{GJ} \qquad (2)$$

And for $\eta < y$

$$C^{\theta\theta}(y,\eta) = \int_0^\lambda \frac{d\lambda}{GJ} \qquad (3)$$

If the distribution of shear flow ϑ (s,λ) caused by a unit torque is given, the constant of torsional J, stated at (2) and (3), can be estimated at any section of the beam. This necessitates knowledge of the wing skin thickness, flange, web and stringer thickness, and so on at each section of the wing, as well as the shear modulus values at each section. According to reference [7], torsional rigidity curve GJ has been calculated and is presented in Fig. 3 (a).

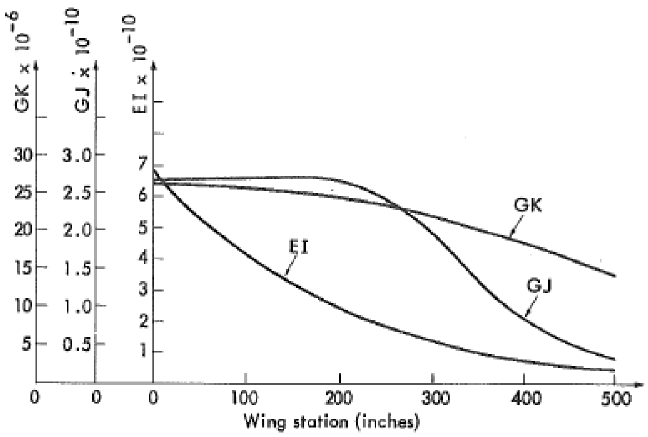

Fig. 3(a). curve of shear stiffness, torsional and bending [7].

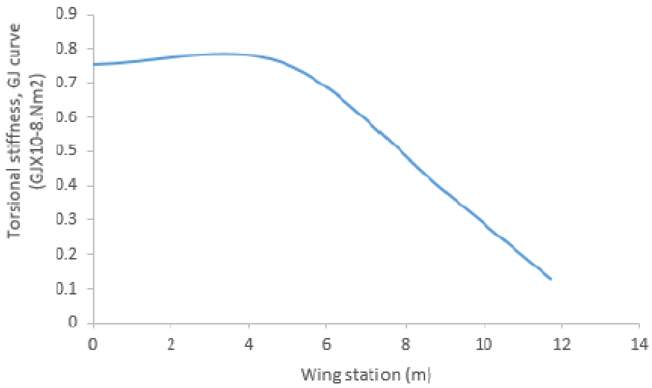

Fig. 3. (b) Curve of torsional stiffness (modified to SI unit) [6]

D. Equilibrium Equations

The following assumes are made for simplicity's sake:

1) Un-swept wings are distinguished via an Elastic-axis that is vertical to the symmetrical plane of aircraft.
2) The wing's chordwise sections maintain rigid; camber bending is to be negligible.

By connecting the rate of twist to the applied torque as previously mentioned, the differential equation of torsional aero-elastic equilibrium of un-swept wing about its elastic axis is represented using equation (1) as follow:

$$\frac{d\theta(y)}{dy} = \theta' = \frac{T}{GJ}$$

we might rewrite this as:

$$\frac{d}{dy}\left(GJ\frac{d\theta}{dy}\right) = \frac{dT}{dy} = -t(y) \qquad (4)$$

(y): distribution of elastic twist.

Take a slender straight wing that is being affected by aerodynamic and inertial forces, as in Fig. 4.

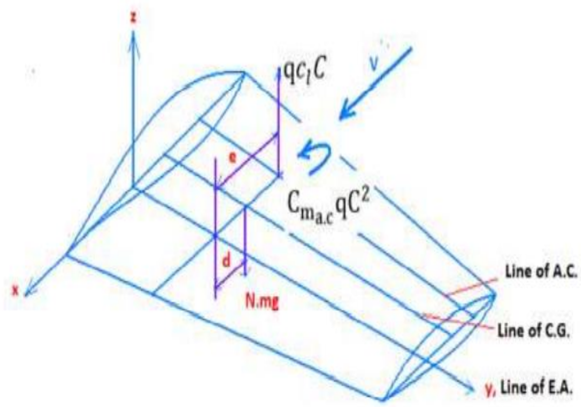

Fig. 4. Straight wing [6].

The utilized torque per unit span t(y) is provided in Fig.4 by:

$$t(y) = qc_lce + qc^2C_{m_{ac}} - Nmgd \qquad (5)$$

Where c_l are the local lift coefficients.

$C_{m_{ac}}$: local coefficients of moment about center of aerodynamic.

mg: weight of wing per unit span.

N is the load factor perpendicular to wing surface. At level flight, N equals 1.

When we combine (4) and (5), we get the following differential equation of equilibrium:

$$\frac{d}{dy}\left(GJ\frac{d\theta}{dy}\right) = Nmgd - qc_l ce - qc^2 C_{m_{ac}} \quad (6)$$

or

$$\frac{d}{C_{m_{ac}}dy}\left(GJ\frac{d\theta}{dy}\right) + qc_l ce = Nmgd - qc^2 \quad (7)$$

$\theta(0) = 0$; $\theta'(l) = 0$. Are the boundary conditions.

By using Castiglione's theorem to the energy equation, the wing's torsional deflection is calculated at any spanwise position y caused by torque t applied at span wise position η.

$$\theta(y) = \int_0^l C^{\theta\theta}(y,\eta)t(y)d\eta \quad (8)$$

When (6) is added to (8), we get:

$$\theta(y) = \int_0^l C^{\theta\theta}(y,\eta)\{[c_l ce + c^2 C_{m_{ac}}]q - Nmgd\}d\eta \quad (9)$$

The angle of attack may be thought of as a superposition of an elastic twist and a rigid angle.

$$\alpha(y) = \alpha^r(y) + \theta(y) \quad (10)$$

Additionally, local coefficient of lift may be expressed as follows:

$$c_l(y) = c_l^r(y) + c_l^e(y) \quad (11)$$

Where $c_l(y)$: local angle of attack determined from zero lift excepting elastic twist.

$c_l^r(y)$: distribution of local coefficient of lift caused by rigid twist, $\alpha^r(y)$.

$c_l^e(y)$: distribution of local coefficient of lift due to elastic twist.

The following differential equation is obtained by substituting (11) in (7):

$$\frac{d}{dy}\left(GJ\frac{d\theta}{dy}\right) + qcec_l^e = -qcec_l^r - qc^2C_{m_{ac}} + Nmgd \quad (12)$$

Similarly, we derive the following integral equation by replacing (11) in (9):

$$\theta(y) = q\int_0^l C^{\theta\theta}(y,\eta)ecc_l^e\, d\eta + f(y) \quad (13)$$

Where:

$$f(y) = \int_0^l C^{\theta\theta}(y,\eta)\left(ecc_l^r q + qc^2 C_{m_{ac}} - Nmgd\right)d\eta$$

In case of actual wing the rigid airloads are zero, thus, $\alpha^r(y)$ and $c_l^r(y)$ are zero.

Equation (13) represents the required governing integral equation.

The integral equation form serves as a convenient basis for numerical solutions of complex practical problems. In both equations differential and integral forms $\theta(y)$ and $c_l{}^e(y)$ are regarded as unknown functions, and all other terms are assumed specified. The problem becomes mathematically determinate as soon as a second relation between the two unknowns is stated; this is supplied by some appropriate choice of aerodynamic theory. the aerodynamic theory is usually assumed to involve a linear relation between incidence and lift distribution which can be represented symbolically by

$$\propto(y) = \Theta[cc_l] \quad (14)$$

Where $\Theta$is a linear operator which operates on the lift distribution $cc_l(y)$ to produce the required incidence distribution $\propto(y)$.

E. *Torsional divergence*

The torsional divergence speed of a three- dimensional wing is determined from the lowest eigenvalue of dynamic pressure q obtained from the homogeneous differential or integral equations of equilibrium . It thus represents that speed at which the wing, arranged so that in the untwisted condition it experiences no aerodynamic moments whatever, is theoretically capable of assuming an arbitrary amount of twist and remaining in neutral equilibrium there under the airloads due to the twist alone. Since the solution to a nonhomogeneous equation becomes infinite for the eigenvalues of the corresponding homogeneous equation, we may conclude that an actual wing (which never can be adjusted so that the rigid airloads are exactly zero) would twist off and be destroyed at its divergence speed. The homogeneous forms of Eqs. (12) and (13) are:

$$\frac{d}{dy}\left(GJ\frac{d\theta}{dy}\right) + qecc_l^e = 0 \quad (15)$$

$$\theta(y) = q\int_0^l C^{\theta\theta}(y,\eta)ecc_l^e\, d\eta \quad (16)$$

Equation (15) or Eq. (16) can be alternatively used together with Eq. (14) to compute the divergence speed. They are both satisfied by the same infinite set of eigenvalues and eigenfunctions. The lowest eigenvalue is the dynamic pressure , q_D, corresponding to torsional divergence. The corresponding eigenfunction θ_D(y) , is the spanwise twist distribution at the divergence speed [7].

F. *Definition of Aerodynamic Lifting Line theory*

Various approximate procedures for solving the linearized problem of the lifting wing. one of these approaches which is particular interest to the aeroelastician is lifting line theory which is introduced in this section. The following Equation is the lifting-line formula , specialized for a rectangular wing whose sectional lift - curve slopes are all 2π [7].

$$\Gamma(y) = 2\pi Ub\left[\propto(y) - \frac{1}{4\pi U}\oint_{-l}^{l}\frac{d\Gamma}{d\eta}\frac{d\eta}{(y-\eta)}\right] \quad (17)$$

As is well known to aeronautical engineers, the general formula was originally derived by replacing the actual vortex sheet with a single , concentrated bound vortex of strength $\Gamma(y)$, from which emanates a wake of trailers having circulation $d\Gamma/dy$ per unit spanwise distance. The running lift $\rho U\Gamma$ at each section was equated to the lift $a_0 0.5\,\rho U^2 c \propto_{eff}$ of a two-dimensional airfoil with chord c and lift-curve slope a_0, working at an effective angle of attack determined by the induced flow pattern there. $\propto_{eff}$ was calculated by subtracting from the geometrical angle $\propto$ (measured from zero-lift attitude) the contribution of the so- called downwash [7].

This downwash, assumed constant along each airfoil chord, is just the downward velocity at the bound vortex line due to the entire vortex sheet. A simple application of the Biot - Savart law therefore yields

$$\propto_{\text{eff}} \cong \propto - \frac{Downwash}{U} = \propto - \frac{1}{4\pi U}\oint_{-l}^{l}\frac{d\Gamma}{d\eta}\frac{d\eta}{(y-\eta)} \tag{18}$$

The equation between the two expressions for running lift, after substitution of Eq. (18) and division by ρU finally reads

$$\Gamma(y) = a_0 U \frac{c}{2}\left[\propto(y) - \frac{1}{4\pi U}\oint_{-l}^{l}\frac{d\Gamma}{d\eta}\frac{d\eta}{(y-\eta)}\right] \tag{19}$$

Since $c/2 = b$, this is obviously consistent with Eq .(17), although here both c and a_0 may be functions of the spanwise coordinate y.

for solving eq(19), The best known is Glauert's Fourier series substitution , which leads in such a natural way to the familiar concepts of elliptic loading and minimum induced drag. A convenient angle variable resembling the one is defined by:

$$\eta = l \times cos\theta \quad , \quad y_i = l \times cos\phi \tag{20}$$

This puts the wing tips at $\theta = 0$ and $\theta = \pi$. Since Γ is known to vanish at both tips, it is taken in the form of a Fourier sine series :

$$\Gamma(y) = \Gamma(\phi) = Ul\sum_{r=1}\bar{A}_r\ sinr\phi \tag{21}$$

The general integral formula comes into use when we substitute eq(21) into the right side of eq(18).

$$\oint_{-l}^{l}\frac{d\Gamma}{d\eta}\frac{d\eta}{(y-\eta)} = -\frac{1}{l}\oint_{0}^{\pi}\frac{d\Gamma}{d\theta}\frac{d\theta}{(cos\phi - cos\theta)}$$

$$= U\oint_{0}^{\pi}\sum_{r=1}\frac{r\bar{A}_r cosr\theta}{(cos\theta - cos\phi)}d\theta$$

$$= \pi U\sum_{r=1} r\bar{A}_r\frac{sinr\phi}{sin\phi} \tag{22}$$

Inserting Eqs. (21) and (22) into Eq . (19) , dividing by Ul and rearranging , we obtain the algebraic equality [7]:

$$\frac{a_0 c}{2l}\propto = \sum_{r=1}\bar{A}_r\left[sinr\phi + \frac{a_0 c}{8l}\frac{rsinr\phi}{sin\phi}\right] \tag{23}$$

G. Difference between the symmetrical and anti-symmetrical divergence modes according to Lifting Line theory

In eq(23), The sine of an odd multiple of ϕ makes a contribution to the spanwise lift distribution which is symmetrical about mid-span , whereas even multiples are antisymmetrical. Hence $\propto$ may be divided into portions $\propto^s$ and $\propto^a$, the former equaling the sum of odd terms on the right of Eq . (23), the latter equaling the sum of even terms. It is customary to solve the two equations thus obtained separately for the constants $\bar{A}_r^s$ and $\bar{A}_r^a$, by requiring them to be satisfied identically at a number of stations along the wing semispan equal to the number of constants needed for adequate convergence in Eq. (21) . In most aeroelastic problems, convenience dictates that these should be the same stations for which structural stiffness properties are known, so that aerodynamic and elastic equations can be combined straightforwardly [7].

In matrix notation, the symmetrical part of eq (23) for such a series of stations is:

$$\frac{1}{2l}[a_0 c]\{\propto^s\} = [sinr\phi]\{\bar{A}_r^s\} + \frac{1}{8l}\left[\frac{a_0 c}{sin\phi}\right][rsinr\phi]\{\bar{A}_r^s\} \tag{24}$$

At each of these stations (or at any other station of interest) the local sectional lift is

$$\rho U\Gamma = 0.5\rho U^2 cc_l \tag{25}$$

therefore eq(21) leads to

$$c(y)c_l(y) = 2l\sum_{r=1}\bar{A}_r\ sinr\phi \tag{26}$$

Or in matrix form for the symmetrical case,

$$\{cc_l{}^s\} = 2l[sinr\phi]\{\bar{A}_r^s\} \tag{27}$$

An aerodynamic matrix relating lift coefficient and angle of attack is derived by eliminating $\bar{A}_r^s$ between (27) and (24)

$$\frac{1}{2l}[a_0 c]\{\propto^s\} = \frac{1}{2l}\left([sinr\phi] + \frac{1}{8l}\left[\frac{a_0 c}{sin\phi}\right][rsinr\phi]\right)[sinr\phi]^{-1}\{cc_l{}^s\} \tag{28}$$

After cancelling and multiplying by $\left[\frac{1}{a_0 c}\right]$, we obtain

$$\{\propto^s\} = \left(\left[\frac{1}{a_0 c}\right] + \frac{1}{8l}\left[\frac{1}{sin\phi}\right][rsinr\phi][sinr\phi]^{-1}\right)\{cc_l{}^s\} = [A^s]\{cc_l{}^s\} \tag{29}$$

The aerodynamic matrix in symmetrical case $[A^s]$ is quite analogous to the more exact relations between local slope and wing loading, except that the present simpler type of wing undergoes no chordwise deformation and leads to a one-rather than a two-dimensional model. The aerodynamic matrix in anti-symmetrical case $[A^a]$ has a form identical to $[A^s]$, except that even rather than odd values of r are involved and no terms need to be included for the mid-span station, where $\propto^a = c_l^a = 0$ [7]. When the semispan is divided into n intervals, the sine matrices in Eq. (29) are illustrated later in part V Section E.

H. Matrix-solution accounting for aerodynamic span effects.

When aerodynamic span effects are taken into consideration, the divergence speed can be found from the lowest eigenvalue of the homogenous integral equation formed from eqs (16) and (14):

$$\Theta[cc_l^e(y)] = q_d\int_0^l C^{\theta\theta}(y,\eta)ecc_l^e d\eta \tag{30}$$

When induction effects are taken into account according to Prandtl's lifting line theory, the functional relation $\Theta[cc_l^e(y)]$ can be derived from the following equation

$$\Gamma(y) = a_0 U\frac{c}{2}\left[\propto(y) - \frac{1}{4\pi U}\oint_{-l}^{l}\frac{d\Gamma}{d\eta}\frac{d\eta}{y-\eta}\right] \tag{31}$$

Introducing $cc_l = 2\Gamma/\text{U}$ and transposing yields

$$\propto(y) = \Theta[cc_l^e(y)] = \frac{cc_l^e(y)}{a_0 c} + \frac{1}{8\pi}\oint_{-l}^{l}\frac{d}{d\eta}(cc_l^e)\frac{d\eta}{y-\eta} \tag{32}$$

Combining eqs (30) and (32),

$$\frac{cc_l^e(y)}{a_0 c} + \frac{1}{8\pi}\oint_{-l}^{l}\frac{d}{d\eta}(cc_l^e)\frac{d\eta}{y-\eta} = q_d\int_0^l C^{\theta\theta}(y,\eta)ecc_l^e d\eta \tag{33}$$

where the local lift coefficient slope a_0 has been assumed constant. Equation (33) is satisfied by an infinite set of eigenvalues q_j ; and eigenfunctions $(cc_l^e)_j$. in this case, the influence of finite span on aerodynamic forces is included. The lowest eigenvalue, q_D, is the dynamic pressure corre-sponding to torsional divergence. The eigenfunctions of Eq. (33) may be either even or odd in y, depending on whether the associated distribution of wing twist is symmetrical or antisymmetrical; these functions correspond, in general, to different divergence speeds [7]. The process of solving Eq. (33) requires considering the symmetrical and anti-symetrical solutions separately. The former is designated by (cc_l^{es}) and the latter by (cc_l^{ea}). Since the most general (cc_l^{e}) can be represented as the sum of (cc_l^{es}) and (cc_l^{ea}) , this sum can be substituted into Eq . (33) and the result separated into two independent parts. When approximation formulas are used to evaluate the integrals, one obtains the following matrix equations:

$$[A^s]\{cc_l^{es}\} = q_d[E]\{cc_l^{es}\} \tag{34}$$

$$[A^a]\{cc_l^{ea}\} = q_d[E]\{cc_l^{ea}\} \tag{35}$$

$$[E] = [C^{\theta\theta}]diag[e]diag[\overline{W}] \tag{36}$$

The aerodynamic matrices $[A^s]$ and $[A^a]$,which represent, respectively, the symmetrical and antisymmetrical relations between $cc_l(y)$and α(y) , are constructed from Eq (33) . An explicit expression for $[A^{\,s}]$ is given by Eq . (29) .

When $[A^{\,s}]$ is substituted into Eq . (34) , multiplication by $[A^{\,s}]^{-1}$ produces a form suitable for matrix iteration to determine the symmetric divergence modes. A similar iteration of Eq. (35) results in the anti- symmetric modes. In each case , the mode corresponding to the lowest value of dynamic pressure q , which is the one obtained without sweeping , is the one of greatest practical interest [7].

IV. THE GEOMETRY OF THE WING UNDER INVESTIGATION

Because the wing in consideration is tapered, the chord changes over the wing span [1].

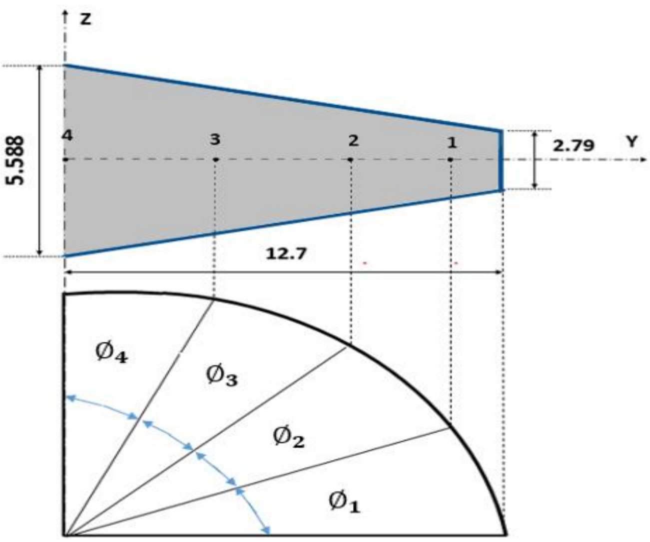

Fig. 5. Wing Multhopp’s stations of the case study [1].
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All dimensions are in meter.
The wing root chord length, C_r = 5.588 m
The wing tip chord length, C_t = 2.794 m
The wing semi span, l = 12.7 m
Aerodynamic center = 25 % chord.
Elastic axis = 35 % chord.

From relation:

$\emptyset_i = \frac{180^\circ}{n+1}$, Assume incompressible flow using lifting line theory (use Multhopp stations with n = 7),
So for Anti-symmetrical case:

$$\emptyset_1 = 22.5^\circ, \emptyset_2 = 45^\circ, \emptyset_3 = 67.5^\circ$$

At any span length, $y_i = l \times cos\phi$
Then:
Point 3 at 4.8601 m from wing root.
Point 2 at 8.9803 m from wing root.
Point 1 at 11.7333 m from wing root.

V. NUMERICAL SOLUTION OF SYMMETRICAL DIVERGENCE MODES OF THE WING

In this paper, the case study is symmetrical divergence modes. Equation (33), which governs the calculation of unswept-tapered wing's divergence speed, has been formulated and is provided in matrices form.

A. Torsional Influence Coefficient Matrix: Numerical Computation.

The obtained matrix of torsional coefficients is square matrix, illustrated as follows:

$$[C^{\theta\theta}] = \begin{bmatrix} C^{11} & C^{12} & C^{13} \\ C^{21} & C^{22} & C^{23} \\ C^{31} & C^{32} & C^{33} \end{bmatrix}$$

These coefficients' values have been calculated from (2) and (3) as follow:

$$C^{\theta\theta} = \int_0^{y_i} \left(\frac{1}{Gj(y)}\right) \times dy$$

Assume torsional rigidity varies as:

$Gj(y) = 71.745 \times 10^6(1 - 0.0394y_i)^4 \; N.\,m^2/rad$

$$C^{\theta\theta} = \int_0^{y_i} \left(\frac{1}{71.745 \times 10^6(1 - 0.0394y_i)^4}\right) dy$$

Therefore,

Flexibility influence coefficients matrix calculated as follows:

$$[C^{\theta\theta}] = \begin{bmatrix} 0.6396 & 0.3188 & 0.1052 \\ 0.3188 & 0.3188 & 0.1052 \\ 0.1052 & 0.1052 & 0.1052 \end{bmatrix} \times 10^{-6} rad/N.m \tag{37}$$

B. Calculation of the matrix for the wing chord

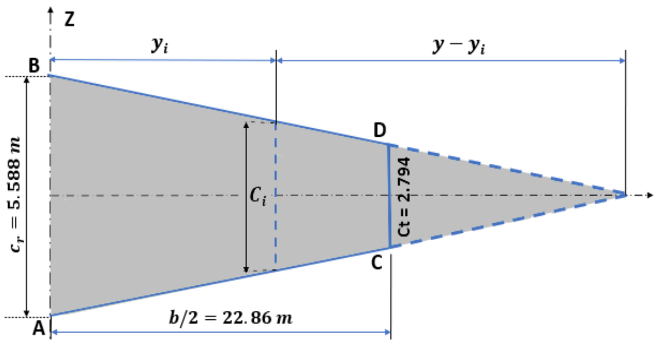

Fig. 6. Geometry of the wing half span platform [1].

Be wing chord at any span y_i to be calculated using the subsequent relationships based on triangle similarity:

$$\frac{C_i}{C_r} = \frac{y - y_i}{y}$$

$$C_i = C_r\left(\frac{y-y_i}{y}\right) = C_r\left(1-\frac{y_i}{y}\right) \qquad (38)$$

If the previous numerical values are substituted, the following will result:

$$C(\mathrm{y}) = 5.588(1 - 0.0394y_i)\ m \qquad (39)$$

The following diagonal matrix is obtained by calculating the chord C_i at stations 1, 2, and 3, then making the results into a matrix:

$$C(y_1) = 5.588(1 - 0.0394 \times 11.7333) \;=\; 3.0067\ m$$
$$C(y_2) = 5.588(1 - 0.0394 \times 8.9803) \;=\; 3.6123\ m$$
$$C(y_3) = 5.588(1 - 0.0394 \times 4.8601) \;=\; 4.5188\ m$$

$$[c] = \begin{bmatrix} 3.0067 & 0 & 0 \\ 0 & 3.6123 & 0 \\ 0 & 0 & 4.5188 \end{bmatrix} m$$

C. *Eccentricity calculation*

The aerodynamic center (A.C.) of the wing is considered to be one-quarter of the chord taken from the leading edge (*0.25×chord, C*), and (E.A.) located at 0.35*C* computed from the leading edge.

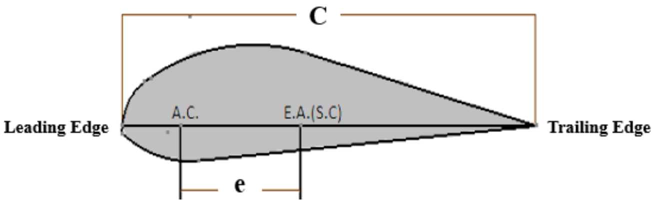

Fig .7. Section of an airfoil illustrating the aerodynamic and shear centers.

The eccentricity, $e = 0.35\mathrm{C} - 0.25\mathrm{C} = 0.1\mathrm{C}$, which is the distance among the elastic axis and aerodynamic center. Because the chord c varies or changes for different parts of the wing, e varies as well.

Therefore,

$$e_i = 0.1 \times C_i \qquad (40)$$
$$\mathrm{e}(c_1) = 0.1 \times 3.0067 = \; 0.3007$$
$$\mathrm{e}(c_2) = 0.1 \times 3.6123 = 0.3612$$
$$\mathrm{e}(c_3) = 0.1 \times 4.5188 = 0.4519$$

That results as diagonal matrix as follows:

$$[e] = \begin{bmatrix} 0.3007 & 0 & 0 \\ 0 & 0.3612 & 0 \\ 0 & 0 & 0.4519 \end{bmatrix}$$

D. *Multhopp's quadratic approach*

Multhopp's approximate quadrature is useful when dealing with functions derived from lifting line theory. utilizing the techniques and formula for a wing semi span $l = b/2$ to anti-symmetrical lift distribution problem yields the diagonal matrix of weighting values [7]. As follow:

The weighted factors matrix for anti-symmetrical case is given by:

$$[\overline{\mathrm{w}}(\emptyset)] = \frac{\pi \times l}{\mathrm{n}+1} \times diag(\sin(\emptyset_1), \sin(\emptyset_2), \sin(\emptyset_3))$$

$$[\overline{W}] = \frac{\pi l}{8}\begin{bmatrix} sin\emptyset_1 & 0 & 0 \\ 0 & sin\emptyset_2 & 0 \\ 0 & 0 & sin\emptyset_3 \end{bmatrix} \qquad (41)$$

$$[\overline{W}] = \begin{bmatrix} 1.9085 & 0 & 0 \\ 0 & 3.5265 & 0 \\ 0 & 0 & 4.6076 \end{bmatrix}$$

E. *Calculate Anti-symmetrical divergence speed of the wing using aerodynamic lifting line theory.*

Anti-symmetrical divergence speed according to lifting line theory is computed from (35):

$$[A^a]\{cc_l^{ea}\} = q_d[E]\{cc_l^{ea}\}$$

The following results is obtained by multiplying the respective matrix values based on (36), which is:

$$[E] = [C^{\theta\theta}]diag[e]diag[\overline{W}]\,.$$

$$[E] = \begin{bmatrix} 0.3670 & 0.4061 & 0.2189 \\ 0.1829 & 0.4061 & 0.2189 \\ 0.0603 & 0.1340 & 0.2189 \end{bmatrix} \times 10^{-6}$$

the $[A^a]$ matrix is given by:

$$[A^a] \;= \mathrm{diag}\left[\frac{1}{\mathrm{a_0 c}}\right] + \frac{1}{8l}[\emptyset^S] \qquad (42)$$

$$[\emptyset^s]=\mathrm{diag}\left[\frac{1}{sin\emptyset_i}\right] \times [\mathrm{n}\times\sin(\mathrm{n}\times \emptyset_i)][sin(n \times \emptyset_i)]^{-1} \qquad (43)$$

$$[\phi^s] = \begin{bmatrix} 2.6131 & 0 & 0 \\ 0 & 1.4142 & 0 \\ 0 & 0 & 1.0824 \end{bmatrix} \times \begin{bmatrix} 1.4142 & 4.0000 & 4.2426 \\ 2.0000 & 0.0000 & -6.0000 \\ 1.4142 & -4.0000 & 4.2426 \end{bmatrix} \times \begin{bmatrix} 0.7071 & 1.0000 & 0.7071 \\ 1.0000 & 0.0000 & -1.0000 \\ 0.7071 & -1.0000 & 0.7071 \end{bmatrix}^{-1}$$

$$[\phi^s] = \begin{bmatrix} 10.4525 & -3.6955 & 0.0000 \\ -2.0000 & 5.6569 & -2.0000 \\ -0.0000 & -1.5307 & 4.3296 \end{bmatrix}$$

$$[A^s] = \begin{bmatrix} 0.0605 & 0 & 0 \\ 0 & 0.0503 & 0 \\ 0 & 0 & 0.0402 \end{bmatrix} + \left(\frac{1}{8 \times 12.7}\right) \times \begin{bmatrix} 10.4525 & -3.8284 & 0.0000 \\ -2.0719 & 5.6569 & -2.3890 \\ -0.0000 & -1.8284 & 4.3296 \end{bmatrix}$$

$$[A^s] = \begin{bmatrix} 0.1634 & -0.0364 & 0.0000 \\ -0.0197 & 0.1060 & -0.0197 \\ -0.0000 & -0.0151 & 0.0828 \end{bmatrix}$$

The matrix to be iterated derives from eq (35) as follow:

$$[cc_l^{ea}] = q[A^a]^{-1} \times [E][cc_l^{ea}]$$

Where:

$$[P] = [A^a]^{-1} \times [E]$$

$$[P] = \begin{bmatrix} 0.2795 & 0.3592 & 0.2016 \\ 0.2463 & 0.4966 & 0.3033 \\ 0.1176 & 0.2520 & 0.3194 \end{bmatrix} \times 10^{-5}$$

$$\frac{1}{q}[cc_l^{ea}] = [P][cc_l^{ea}] \qquad (44)$$

When we apply matrix iteration ($q \to q_D$).

$$\frac{100}{q_D}[cc_l^{ea}] = 100 \times [P][cc_l^{ea}]$$

$$\lambda = \frac{100}{q_D}$$

$$\lambda[cc_l^{ea}] = 100 \times [P][cc_l^{ea}] \qquad (45)$$

Applying matrix iteration to eq (45) and assume firstly:

$$[cc_l] = \begin{Bmatrix} 1 \\ \cdot \\ \cdot \\ 1 \end{Bmatrix}$$

Finally gives the following after complete iteration:

$$\lambda = 8.8139 \times 10^{-4} \begin{bmatrix} 0.8032 \\ 1.0000 \\ 0.6165 \end{bmatrix} \qquad (46)$$

Anti-symmetrical mode shape are given by the column matrix in (46), and the Anti-symmetric divergence speed is obtained from (47) as follows:

$$v_d = \sqrt{\frac{2\times q_d}{\rho}} \qquad (47)$$

Where, $\lambda = 8.8139 \times 10^{-4}$

Then, $q_d = \frac{100}{8.8139\times10^{-4}} = 1.1346 \times 10^5$

For the case of flight at sea level $\rho = 1.225$ kg/m^3

$$v_d = \sqrt{\frac{2\times1.1346\times10^5}{1.225}} \quad , \qquad v_d = 430.3905\ m/sec.$$

CONCLUSIONS

Aerodynamic span effects are taken into consideration throughout this research to determine the antisymmetrical divergence speed of unswept-tapered wing according to lifting line theory from the lowest eigenvalue of the governing integral equation (33). The lowest eigenvalue, q_D, is the dynamic pressure corre-sponding to torsional divergence. In this study, The eigenfunctions of Eq. (33) were even in y. So, the attained distribu-tion associated with the wing twist is antisymmetric.

In short, the ordinates to the antisymmetrical distribution of the wing spanwise twist at the divergence speed are given by the column matrix of eq (46), and the antisymmetrical divergence speed is obtained from (47). Anti-symmetrical divergence speed of the wing has been found in this work for the case of flight at sea level.

The outcome obtained from the similar case study presented in reference [7] supported the results attained in this paper. In comparing the attained result with reference [7], we see that when lifting - line theory results (symmetric and antisymmetric) are compared, the antisymmetric divergence speed is higher than the symmetric. This is generally true for straight wings. the symmetric divergence speed in this study is 413.0558 m/sec after replace an even multiple of ϕ in eq (23) by odd values.

The obtained results will aid modern aircraft designers in their understanding of wing instability in steady motion.

APPENDIX

A. Calculation Anti-symmetrical divergence speed using aerodynamic lifting line theory.

```
clc
clear all
% Anti-symmetric
phi = [(pi/8) (2*pi/8) (3*pi/8)]
A = eye(3)
A(1,1)=1/sin(phi(1));
A(2,2)=1/sin(phi(2));
A(3,3)=1/sin(phi(3));
disp(A)
B = [2*sin(2*phi(1)) 4*sin(4*phi(1))
6*sin(6*phi(1));2*sin(2*phi(2))
4*sin(4*phi(2))
6*sin(6*phi(2));2*sin(2*phi(3))
4*sin(4*phi(3)) 6*sin(6*phi(3))]
C2 = [B(:,1)/2  B(:,2)/4  B(:,3)/6 ]
Geo = A*B*inv(C2)
y = 12.7*cos(phi)
c=5.588*([1 1 1]-(y/25.40))
diag(c)
e=0.1*c
diag(e)
W=(pi*12.7/8)*eye(3)*[sin(phi(1))
sin(phi(2)) sin(phi(3))]'
diag(W)
CC = [6.3956 3.1880 1.0515;3.1880 3.1880
1.0515;1.0515 1.0515 1.0515]*10^-7
EE=CC*diag(e)*diag(W)
D = [(1/(5.5*c(1))) (1/(5.5*c(2)))
(1/(5.5*c(3)))]
Aero = diag(D)+(1/8/12.7)*Geo
P=inv(Aero)*EE
Ccl1=[1 1 1]'
for i=1:15
lamdaCcl= 100*P*Ccl1
lamda = max(lamdaCcl)
Ccl1=lamdaCcl/lamda
end
lamda
Ccl1
qD=100/lamda
VD=sqrt(2*qD/1.225)
```

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